Chapter 4 – Hypothesis Testing

Note Title 15/01/2010

We can use statistics to try to test claims such as

"reducing speed limits decreases the number of accidents"

We cannot be 100% certain that such a claim is true, because the data collected would be subject to chance variations in any case. However we can test such a claim at a certain SIGNIFICANCE LEVEL (e.g. 5%) – ie, we would say that there is only a 5% chance of the data occuring due to random variation, so we can be 95% certain that the claim is justified.

Stages in carrying out a hypothesis test

We will test the hypothesis
"Frannie can tell the difference between Coke and Diet Coke."

① State the Null Hypothesis (H_0) This should always be 'any variation is due to chance'

H_0: "Frannie cannot tell the difference" ie, $p(\text{success}) = \frac{1}{2}$

② Based on H_0, work out the probability distribution of X (the number of successful guesses).

We will repeat the test 13 times, so X = the number of successes in 13 trials.

So $X \sim Bin\left(13, \frac{1}{2}\right)$

(3) State the Alternative Hypothesis (H_1)

H_1 : "Frannie can tell which is Coke and which is Diet Coke"

ie, $p(\text{success}) > \frac{1}{2}$.

The wording of H_1 will determine whether we are using a "one-tailed" or "two-tailed" test.

The above indicates a one-tailed test ie, X must be high to justify H_1

[If we had chosen

H_1: "Frannie can tell the difference (but not necessarily which is which)"

ie, $p(\text{success}) \neq \frac{1}{2}$

this would be a "two-tailed" test and a very high or very low value of X would justify H_1]

(4) Find the "critical region" for X

If H_0 is true, $X \sim Bin(13, \frac{1}{2})$

Value of X	prob	cum prob	
13	$\left(\frac{1}{2}\right)^{13} = 0.0001$		
12	${}^{13}C_{12} \left(\frac{1}{2}\right)^{12}\left(\frac{1}{2}\right) = 0.0016$	0.0017	
11	${}^{13}C_{11} \left(\frac{1}{2}\right)^{11}\left(\frac{1}{2}\right)^{2} = 0.0095$	0.0112	
10	${}^{13}C_{10} \left(\frac{1}{2}\right)^{10}\left(\frac{1}{2}\right)^{3} = 0.0349$	0.0461	$< 5\%$
9	${}^{13}C_{9} \left(\frac{1}{2}\right)^{9}\left(\frac{1}{2}\right)^{4} = 0.0873$	0.1334	$> 5\%$

At a 5% significance level, the critical region is $X \geq 10$.

(because if H_0 is true, $p(X \geq 10) < 5\%$
but $p(X \geq 9) > 5\%$.)

(5) Look at the test statistic and draw the appropriate conclusion.

Test statistic is $X = 11$

This lies in the critical region, so we have sufficient evidence to reject H_0 in favour of H_1:

Frannie can tell which is Regular and which is Diet Coke.

[If the test statistic was not in the critical region, we would say that there was insufficient evidence to reject H_0.]