Note Title 04/12/2009

# Chapter 4 – Sampling Distributions

We take a sample in order to find an estimate of a population PARAMETER (eg mean, proportion etc).

Notation:

We usually use Greek letters for population parameters

e.g. $\mu$ for mean
$\sigma$ for standard deviation

and English letters for statistics calculated from a sample e.g. $\bar{x}$ for mean of a sample
$s$ for s.d of a sample.

A STATISTIC has to be calculable from the values in the sample – it can't involve population parameters (which are usually not known)

e.g. $\sqrt{\frac{\sum x^2}{n} - \bar{x}^2}$ is a statistic

but $\sqrt{\frac{\sum x^2}{n} - \mu^2}$ is not a statistic

If we took a different sample from a population, we would (probably) get a different estimate of the population parameter. The distribution of all values of the statistic which could be found from all possible samples is called the SAMPLING DISTRIBUTION of the statistic.

We can illustrate the concepts above using a small artificial population.

Example Given a population $\{1, 1, 1, 2, 2, 3\}$

(a) Population PARAMETERS mean $\mu = \frac{10}{6} = 1\frac{2}{3}$

proportion of '1's $(\pi) = \frac{1}{2}$

(b) Since we (supposedly) can't know the above facts for the whole population, we decide to take a sample of 3 from the population to estimate these parameters.

List all the possible samples, with the probability of obtaining each one. (Sampling with replacement)

| | |
|---|---|
| 1 1 1 | $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$ |
| 1 1 2 | $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{3} \times 3 = \frac{1}{4}$ |
| 1 1 3 | $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{6} \times 3 = \frac{1}{8}$ |
| 1 2 2 | $\frac{1}{2} \times \frac{1}{3} \times \frac{1}{3} \times 3 = \frac{1}{6}$ |
| 1 2 3 | $\frac{1}{2} \times \frac{1}{3} \times \frac{1}{6} \times 6 = \frac{1}{6}$ (3! = 6 arrangements) |
| 1 3 3 | $\frac{1}{2} \times \frac{1}{6} \times \frac{1}{6} \times 3 = \frac{1}{24}$ |
| 2 2 2 | $\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} = \frac{1}{27}$ |
| 2 2 3 | $\frac{1}{3} \times \frac{1}{3} \times \frac{1}{6} \times 3 = \frac{1}{18}$ |
| 2 3 3 | $\frac{1}{3} \times \frac{1}{6} \times \frac{1}{6} \times 3 = \frac{1}{36}$ |
| 3 3 3 | $\frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{216}$ |

(c) Find the sampling distribution of the mean of samples of size 3.

| $\bar{x}$ | $p(\bar{X} = \bar{x})$ | |
|---|---|---|
| $1$ | $\frac{1}{8}$ | (111) |
| $1\frac{1}{3}$ | $\frac{1}{4}$ | (112) |
| $1\frac{2}{3}$ | $\frac{1}{8} + \frac{1}{6} = \frac{7}{24}$ | (113, 122) |
| $2$ | $\frac{1}{6} + \frac{1}{27} = \frac{11}{54}$ | (123, 222) |
| $2\frac{1}{3}$ | $\frac{1}{24} + \frac{1}{18} = \frac{7}{72}$ | (133, 223) |
| $2\frac{2}{3}$ | $\frac{1}{36}$ | (233) |
| $3$ | $\frac{1}{216}$ | (333) |

(d) Find the sampling distribution of the proportion of '1's in the sample.

| $p$ | $p(P = p)$ |
|---|---|
| $0$ | $\frac{1}{27} + \frac{1}{18} + \frac{1}{36} + \frac{1}{216} = \frac{1}{8}$ |
| $\frac{1}{3}$ | $\frac{1}{6} + \frac{1}{6} + \frac{1}{24} = \frac{9}{24} = \frac{3}{8}$ |
| $\frac{2}{3}$ | $\frac{1}{4} + \frac{1}{8} = \frac{3}{8}$ |
| $1$ | $\frac{1}{8} = \frac{1}{8}$ |

$$E(P) = \left(0 \times \frac{1}{8}\right) + \left(\frac{1}{3} \times \frac{3}{8}\right) + \left(\frac{2}{3} \times \frac{3}{8}\right) + \left(1 \times \frac{1}{8}\right) = \frac{1}{2}$$

Note that this is equal to the proportion of '1's in the population.