Using One Distribution as an Approximation to Another

Note Title 28/09/2009

We can sometimes avoid lengthy calculations in this way, provided certain conditions are met.

Poisson approximation to Binomial.

Providing n is large and p is small, we can approximate $Bin(n,p)$ by a Poisson distribution with the same mean:

$$\boxed{\begin{array}{l} \text{If } \quad X \sim Bin(n,p) \quad \text{with } n > 50 \text{ and } p \leq 0.1 \\ \text{then } \quad X \overset{\text{approx}}{\sim} Po(np) \end{array}}$$

Example A machine produces items of which on average 2% are defective. Find the probability that in a sample of 500 there are more than 15 defectives.

Let $X =$ no of defective items $\Rightarrow X \sim Bin(500, 0.02)$

$\Rightarrow X \overset{\text{approx}}{\sim} Po(10)$

$$\begin{aligned} P(X > 15) &= 1 - P(X \leq 15) \\ &= 1 - 0.9513 \\ &= \underline{\underline{0.0487}} \end{aligned}$$

[Note that if p is small, then q is close to 1, so for the Binomial distn,

$$Var(X) = npq \approx np = E(X)$$

which supports the suggestion that the Poisson dist'n will be a good approximation.]

Normal Approximation to Binomial

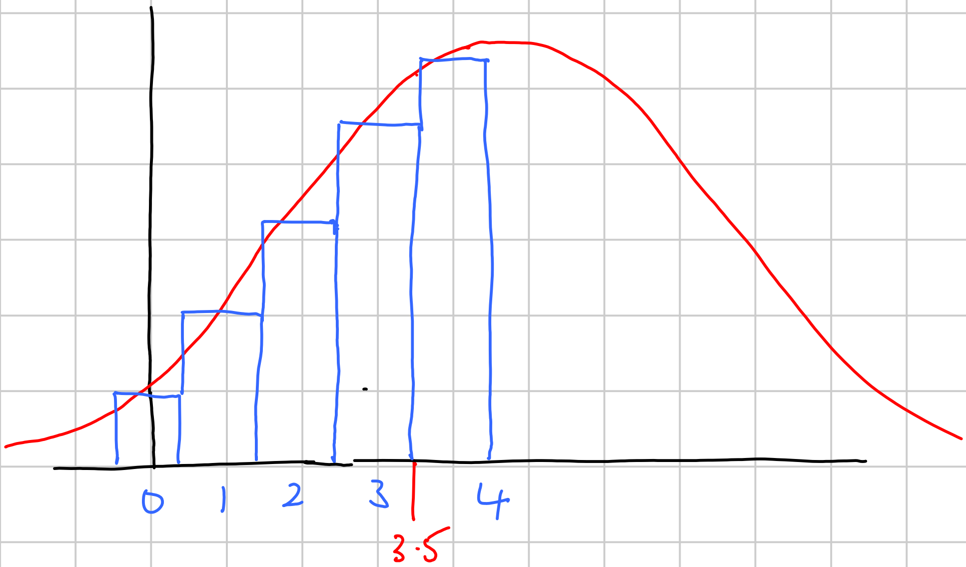
If p is close to $\frac{1}{2}$, $Bin(n, p)$ is fairly symmetrical and the Normal distribution is a good approximation even for relatively small values of n.

The further p is from $\frac{1}{2}$, the larger n needs to be for a good Normal approximation.

Summarising,

$$\boxed{\begin{array}{l} Bin(n, p) \text{ can be approximated by } N(\overset{\mu}{np}, \overset{\sigma^2}{npq}) \\ \text{provided that } np \geqslant 10 \text{ and } nq \geqslant 10 \end{array}}$$

Because we are using a continuous distribution (Normal) to approximate a discrete distribution (Binomial) we have to use a CONTINUITY CORRECTION



Binomial		Normal
$p(X \le 3)$	$\rightarrow$	$p(X \le 3.5)$
$p(X < 3)$	$\rightarrow$	$p(X \le 2.5)$
$p(X \ge 3)$	$\rightarrow$	$p(X \ge 2.5)$
$p(X > 3)$	$\rightarrow$	$p(X \ge 3.5)$

Example In the manufacture of transistors, it is known that 4% of those produced will be faulty. A batch of 500 transistors is inspected. Find the probability that it contains over 30 faulty transistors.

X = no of faulty transistors.

$X \sim \text{Bin}(500, 0.04)$

[$np = 20$ and $nq = 480$ so can use normal approx.]

$$X \overset{\text{approx}}{\sim} N(20, 19.2)$$

$$p(X > 30) \ (\text{Bin}) = p(X > 30.5) \ (\text{Normal})$$

$$= p\left(Z > \frac{30.5 - 20}{\sqrt{19.2}}\right)$$

$p(X > x) \Rightarrow p\left(Z > \frac{x-\mu}{\sigma}\right)$

$$= p(Z > 2.40)$$
$$= 1 - p(Z < 2.40)$$
$$= 1 - 0.9918$$
$$= \underline{\underline{0.0082}}$$

p 81 Ex 3B Q 1, 3 abc, 4 ab, 6, 9

Normal Approximation to Poisson

If λ is large, a Poisson distribution can be approximated by a Normal distribution

$$\boxed{\begin{array}{c}\text{Provided } \lambda > 10 \\ X \sim Po(\lambda) \Rightarrow X \overset{\text{approx}}{\sim} N(\lambda, \lambda)\end{array}}$$

As for the Binomial, we have to use a CONTINUITY CORRECTION.

Example A typist makes on average 3 errors per page. Assuming the number of errors follows a Poisson distribution, find the probability that

(a) He makes more than 20 errors in a 5-page document

Let X = no of errors in 5 pages

$X \sim Po(15) \Rightarrow X \overset{\text{approx}}{\sim} N(15, 15)$

$$\overset{(Po)}{P(X > 20)} = \overset{\text{Normal}}{P(X > 20.5)}$$

$$= P\left(Z > \frac{20.5 - 15}{\sqrt{15}}\right)$$

$$= P(Z > 1.42)$$

$$= 1 - P(Z < 1.42)$$

$$= 1 - 0.9222$$

$$= \underline{\underline{0.0778}}$$

P 81 Ex 3B Q 7, 11, 13