S2 Chapter 1 – Some Discrete Probability Distributions

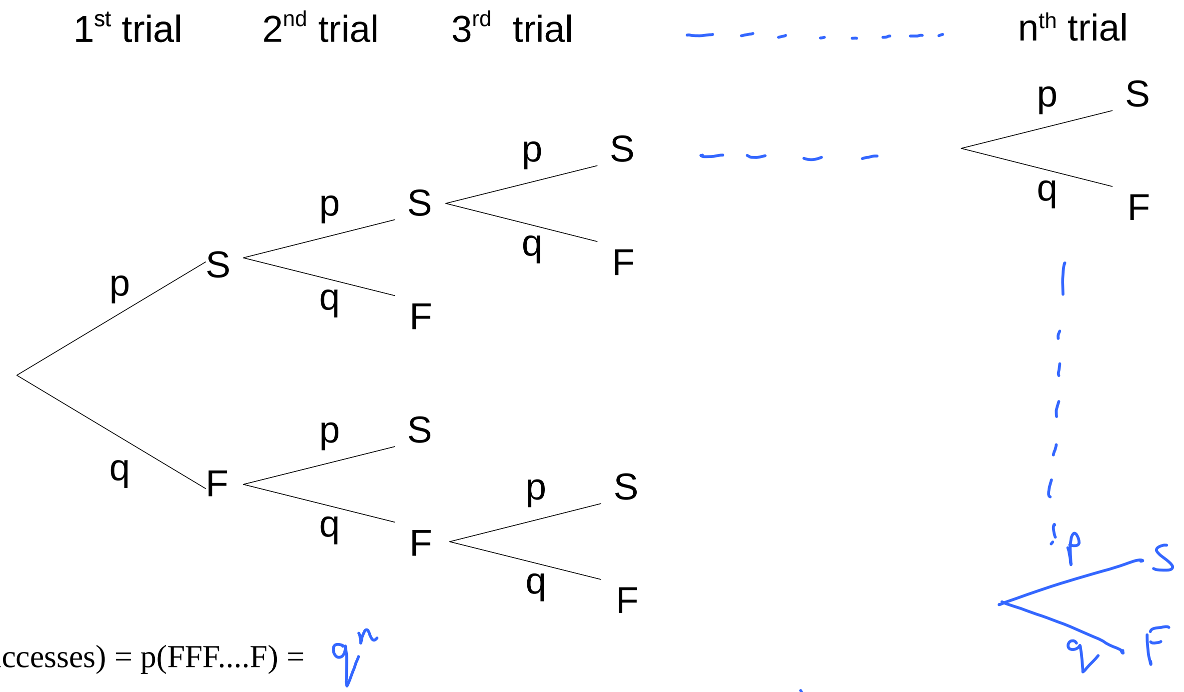
A probability distribution is a way of modelling a real-life situation. Some PDs are especially useful because they are applicable to a wide range of real-life situations. These are studied and various properties about them are established.

The Binomial Distribution

Conditions which give rise to a Binomial Distribution:

- A fixed number (n) of trials is carried out
- For each trial we are interested in 2 outcomes which we can call "success" and "failure". "Success" has probability p and "failure" has probability q = 1 – p
- The random variable X is the number of successes in the n trials. So X can take values {0, 1, 2, …, n}
- The trials are independent, so that the probability of a success remains constant at each trial, no matter what the outcome of the other trials.

The probabilities for a binomial distribution can be found using a tree diagram:



p(0 successes) = p(FFF....F) = q^n

p(1 success) = p(FFF...FS) + p(FFF...SF) + … + p(SFF...F) = $q^{n-1}p \times {}^nC_1$

p(2 successes) = p(FFF...FSS) + p(FFF...SFS) … + p(SSF...FF) = $q^{n-2}p^2 \times {}^nC_2$

p(3 successes) = $q^{n-3}p^3 \times {}^nC_3$

Summary of information for the Binomial Distribution:

x	p(X=x)
0	q^n
1	${}^nC_1\, q^{n-1}p$
2	${}^nC_2\, q^{n-2}p^2$
…	
r	${}^nC_r\, q^{n-r}p^r$
…	
n	p^n

X ~ Bin(n, p) (**parameters** are n and p)

E(X) = np

Var(X) = npq

(For proof of these results see p20 of S2 textbook. You do not need to know the proofs.)

Example A production line produces transistors of which 2% are faulty. A sample of 40 transistors is taken. Find the probability that it contains

(a) 3 faulty transistors

(b) at most 2 faulty transistors

(c) more than 3 faulty transistors

Let X = the no of faulty transistors

Then $X \sim Bin(40, 0.02)$

(a) $$p(X=3) = {}^{40}C_3 (0.98)^{37} (0.02)^3$$
$$= 0.037429\ldots$$

(b) $$p(X \leq 2) = p(X=0) + p(X=1) + p(X=2)$$
$$= (0.98)^{40} + {}^{40}C_1 (0.98)^{39} (0.02) + {}^{40}C_2 (0.98)^{38} (0.02)^2$$
$$\approx 0.954329\ldots$$

(c) $$p(X > 3) = 1 - p(X \leq 3)$$
$$= 1 - p(X = 0, 1, 2 \text{ or } 3)$$
$$= 1 - ((a) + (b))$$
$$= \underline{\underline{0.008241}}$$

p17 Ex 1C Q 1, 7, 8, 9, 10, 12, 13, 14

Using cumulative probability tables

These tables give $p(X \quad x)$ for each x.

Examples

(1) $X \sim Bin(10, 0.25)$. Find:

(a) $p(X \leq 5) = 0.9803$

(b) $p(X \geq 5) = 1 - p(X \leq 4)$
$= 1 - 0.9219$
$= 0.0781$

(c) $p(4 \leq X \leq 7) = \underline{p(X \leq 7)} - \underline{p(X \leq 3)}$

0 1 2 3] 4 5 6 7] 8 9 10

$= 0.9996 - 0.7759$
$= 0.2237$

(d) $p(4 < X < 7) = \underline{p(X \leq 6)} - \underline{p(X \leq 4)}$

0 1 2 3 4] 5 6] 7 8 9 10

$= 0.9965 - 0.9219$
$= 0.0746$

(2) (a) If $X \sim Bin(10, 0.6)$, find (a) $p(X < 3)$
(b) $p(3 \leq X \leq 6)$

Let X' = the number of failures
$X' \sim Bin(10, 0.4)$

(a) $p(X < 3) = p(X' > 7)$ ← Swap the inequality, 3 becomes 10 − 3
$= 1 - p(X' \leq 7)$
$= 1 - 0.9877$
$= 0.0123$

(b) $p(3 \leq X \leq 6) = p(4 \leq X' \leq 7)$ ← 10 − 3 = 7, 10 − 6 = 4
$= p(X' \leq 7) - p(X \leq 3)$
$= 0.9877 - 0.3823 = 0.6054$

Example of a "two stage" problem

A certain scratch card has stars and circles distributed randomly in a 5 by 4 grid, with on average 40% of the symbols being stars. For example:

∗	○	∗	○	○
○	∗	○	∗	○
∗	∗	○	○	∗
○	○	∗	○	○

A contestant scratches off 5 spaces and wins if they get 4 or more stars.

(a) Find the probability of winning if you buy one of these cards.
(b) Ten people each buy one of these cards. Find the probability that two or more of them win.

(a) Let X be the number of stars which we reveal

Then $X \sim Bin(5, 0.4)$

$$P(X \geq 4) = 1 - P(X \leq 3)$$
$$= 1 - 0.9130$$
$$= \underline{\underline{0.0870}}$$

(b) Let Y be the number of people who win

Then $Y \sim Bin(10, 0.087)$ $\quad (p = 0.087, \; q = 0.913)$

$$P(Y \geq 2) = 1 - P(Y \leq 1)$$
$$= 1 - P(Y = 0 \text{ or } 1)$$
$$= 1 - \left[(0.913)^{10} + {}^{10}C_1 (0.913)^9 (0.087)\right]$$
$$= \underline{\underline{0.2141}}$$