PERMUTATIONS AND COMBINATIONS

Permutations

Example A club of 10 people needs to choose a chair, a secretary and a treasurer. In how many ways can this be done.

Answer: 10 ways of choosing chair
9 " " " secretary (once chair chosen)
8 " " " treasurer

So $10 \times 9 \times 8 = \underline{\underline{720 \text{ ways}}}$

This could also be written as $\dfrac{10!}{7!}$ $\left(\dfrac{10\times 9\times 8\times \cancel{7}\times \cancel{6}\times \cancel{\cdots}\times \cancel{1}}{\cancel{7}\times\cancel{6}\times\cancel{5}\times\ldots\ldots\cancel{1}}\right)$

The symbol for this is ${}^{10}P_3$ (the number of ways of permuting 3 objects chosen from 10).

Generalising, $${}^{n}P_r = \underbrace{n\times(n-1)\times(n-2)\times\cdots\cdots}_{r \text{ terms}}$$

$$= \frac{n!}{(n-r)!}$$

Combinations

Example A group of 10 people wishes to choose 3 of them to go and buy some cakes. In how many ways can this be done?

This is <u>not</u> the same as the previous example, but it is related.

The 720 possibilities above include, for example:

Chair	Sec	Treas
Asya	Isobel	Susan
Isobel	Susan	Asya
	etc	

(there are $3 \times 2 \times 1 = 6$ ways of arranging these 3 names)

But these only count as 1 possible group of 3 people

So there are $\frac{720}{6} = \underline{\underline{120}}$ ways of choosing 3 people from 10.

This could also be written as $\frac{10!}{7! \times 3!} = 120$

The symbol for this is ${}^nC_r$ which means the number of ways of choosing r objects from n.

Generalising,
$$ {}^nC_r = \frac{n!}{r!(n-r)!} $$

More examples

(1) A small child has a set of building bricks of two colours, lilac and red. She plays at arranging them in a line.

(a) If she makes a line of 5 bricks, 2 red and 3 lilac, in how many ways would she arrange them?

L	R	L	L	R

First she can choose where to place the two red bricks in ${}^5C_2 = \underline{\underline{10 \text{ ways}}}$

Then the lilac bricks fill the remaining places.

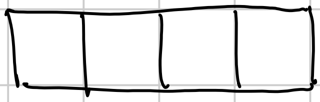
(Alternatively she could start by placing the lilac bricks, in $^5C_3 = 10$ ways. Then the red bricks fill the remaining places.

Since these must give the same answer, it must be true that $^5C_2 = {}^5C_3$

and, e.g $^{10}C_4 = {}^{10}C_6$ etc

Generalising $^nC_r = {}^nC_{n-r}$)

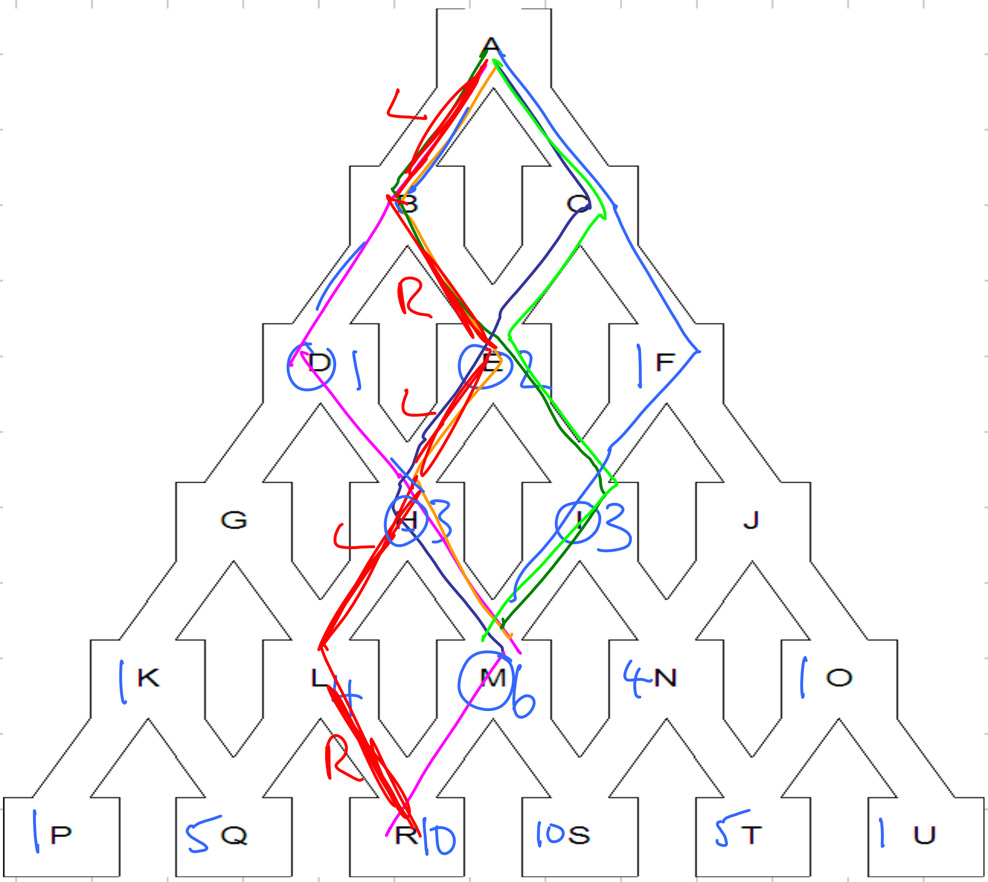
(b) If she has 4 bricks, 3 lilac and 1 red :—



Place the red first in $^4C_1 = \underline{\underline{4 \text{ ways}}}$. (or $^4C_3 = 4$)

(note that $^nC_1 = n$)

② A ball is dropped into the 'Quincunx' below



(a) If the ball ends at R, where was it immediately beforehand? Answer: L or M

(b) How many ways can the ball get to L? 4

(c) " " " " " " " " M? 6

(d) " " " " " " " " R?

Answer $4 + 6 = \underline{\underline{10}}$

Since this method can apply to the number ways of reaching any box, the numbers are given by Pascal's Triangle.

(e) What is the connection between this and the child's bricks?

If we interpret L and R as Left and Right then each arrangement LRLLR, etc corresponds to one route through the Quincunx to box R.