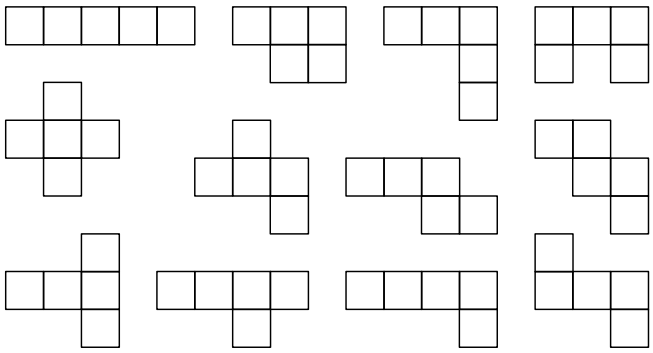
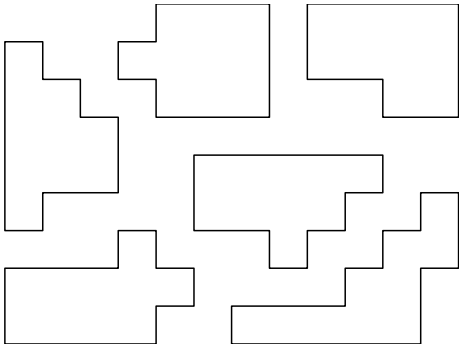
Shapes Puzzle 1

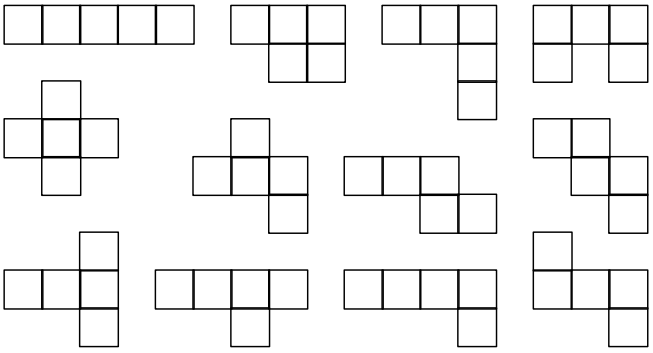
The twelve pentominoes are shown on the left. On the right, they have been placed together in pairs. Can you show which two pentominoes have been used to make each shape? (Each pentomino is only used once.) The solution can be obtained logically, without any trial and error. Try to explain how you find your solution.

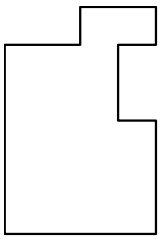




Shapes Puzzle 2

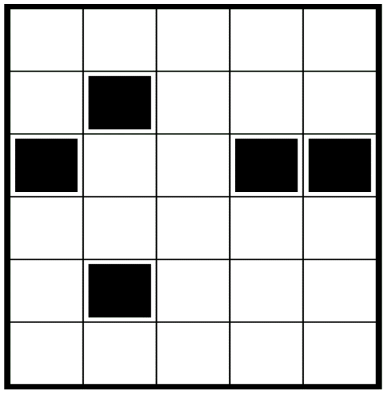
Since pentominoes proved very popular last month, here is another puzzle. The twelve pentominoes are shown on the left. On the right is a shape which can be formed from four pentominoes. Divide the 12 shapes into 3 groups of 4, and fit each group together to make the shape on the right (so you will end up with three copies of this shape, which between them use the 12 pentominoes. Try to explain how you find your solution. (Hint: look for key shapes which can only fit in certain places.)



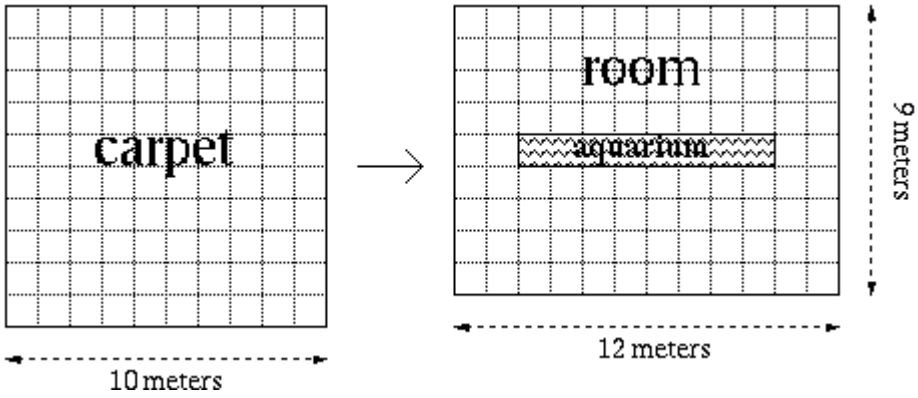


Shapes Puzzle 3

Paint 10 more squares black so that the 5x6 unit rectangle on the right is divided into two pieces: one black and the other white, each with the same size and shape.



Shapes Puzzle 4



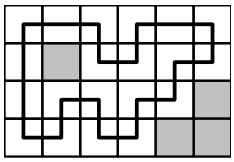
I have a piece of carpet 10m square.

I want to use it to carpet my lounge, which is 12m by 9m, but has a fixed aquarium 1m by 8m in the centre, as shown.

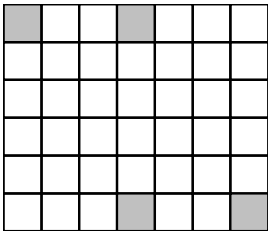
Show how I can cut the carpet into just two pieces which I can then use to carpet the room exactly.

[Hint: The cut is entirely along the 1m gridlines shown.]

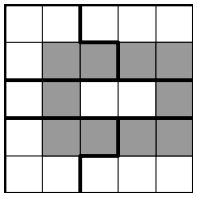
Shapes Puzzle 5



Can you draw a continuous line which passes through every square in the grid on the right, except the squares which are shaded in? (The grid on the left has been done for you as an example.) Try to explain the main steps towards your solution, even if you can't explain every detail.



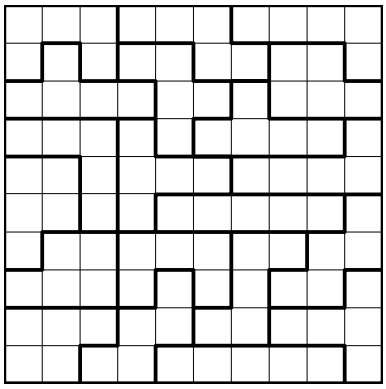
Shapes Puzzle 6



This puzzle combines the path theme of last month with the pentominoes from previous months. Can you shade in squares on the diagram on the right, so that you form a closed path which follows the following rules:

- ✔ The path consists of squares joined edge to edge.
- ✔ The path never touches itself, not even diagonally.
- ✔ In each pentomino, exactly two squares are shaded in.

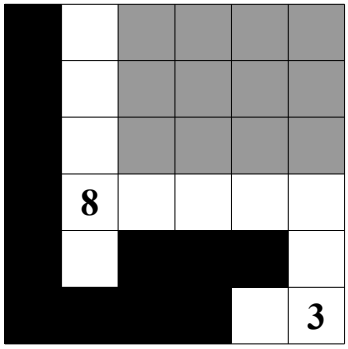
[The diagram on the left gives an example of a correctly formed path]



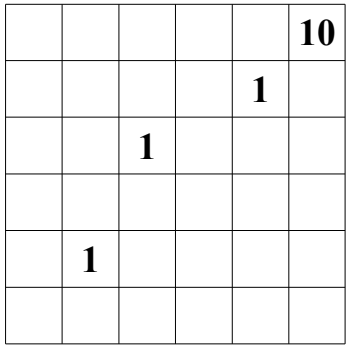
Shapes Puzzle 7

Show how a regular hexagon can be dissected into 8 congruent pieces.

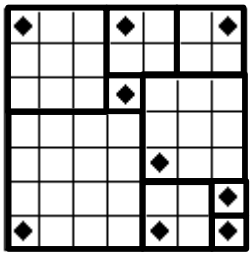
Shapes Puzzle 8



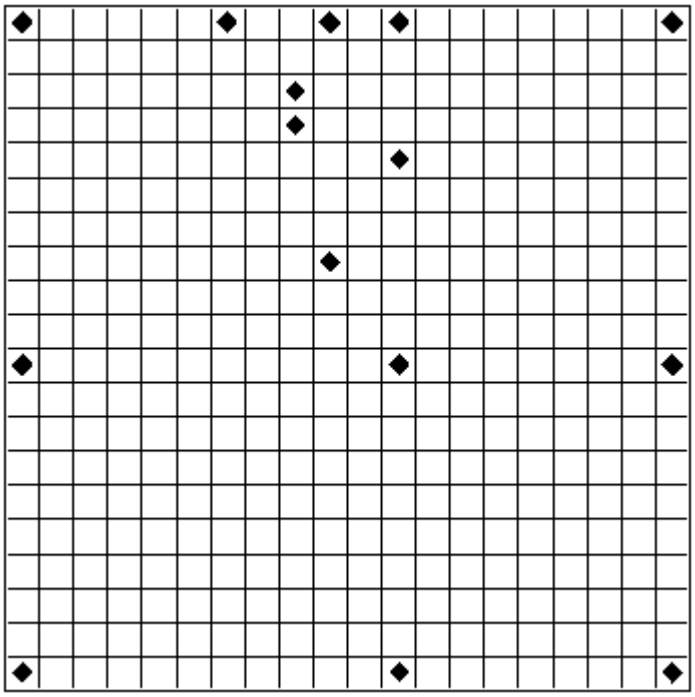
The "house" on the left has been divided into 3 equal sized "rooms" (containing 12 squares each) so that the number in each square indicates how many other squares can be seen from that point horizontally or vertically. Divide the "house" on the right in the same way.



Shapes Puzzle 9



The grid on the left has been divided into squares in such a way that each square contains exactly one diamond. Divide up the grid on the right in the same way. Explain the logic that you use.



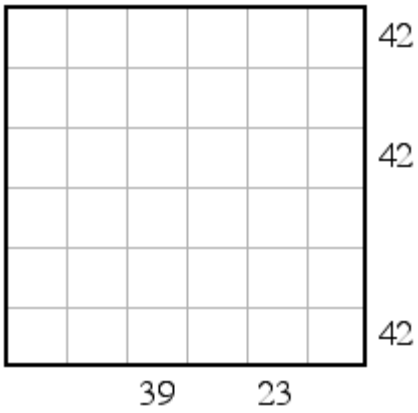
Shapes Puzzle 10

3	3	3	7	7	5	
6	6	1	7	5	5	30
6	6	2	7	7	5	
6	6	2	7	7	5	
8	8	8	8	8	8	48
8	8	4	4	4	4	
		20			32	

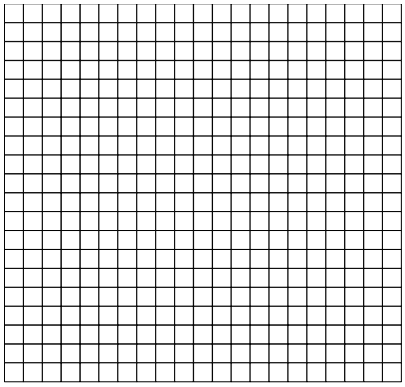
The square on the left has been filled with eight pieces: one monomino (single square), one domino, one tromino, etc, up to one octomino (shape made of 8 squares).

Each small square inside each polyomino has the corresponding number written in it (for example the five squares of the pentomino each contain the number 5). Some of the rows and columns have the total of their of their numbers shown.

Can you fill the square on the right with one polyomino of each size from 1 to 8 so that the the row and column totals shown are correct?



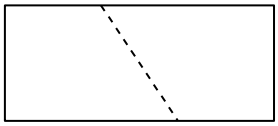
Shapes Puzzle 11



I want to cover this grid of 21 by 20 small squares with square tiles of various sizes. The tiles are non-overlapping, and with integer sides. I could use one big 20-by-20 tile, and 20 small 1-by1 tiles down the edge. That would be a total of 21 tiles. But I want to use as **few** tiles as possible. What is the smallest number of tiles I can use, and what's the exact arrangement?

Shapes Puzzle 12

A sheet of paper 10cm by 20cm is folded so that diagonally opposite corners meet.
What is the exact length of the fold (shown by a dotted line on the diagram)?



[Hints:

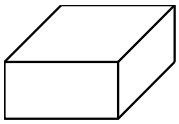
- You might want to experiment with a piece of paper first.
- You need to know about similar triangles and Pythagoras' Theorem for this puzzle.]

Shapes Puzzle 13

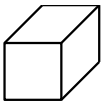
Suppose we have three infinite straight lines drawn on a plane (a flat surface), such that no three pass through the same point.
How many crossing points could there be?
The possibilities are: 0 (three parallel lines), 2 (two parallel lines and one line across them), or 3 (no lines parallel).
[Note that 1 cannot occur because of the restriction that no three lines pass through the same point.]

Given six infinite straight lines drawn on a plane, no three through the same point, what are the possible numbers of crossing points?
Explain the logic you use.

Shapes Puzzle 14



6 of these



3 of these

"These parcels must fit somehow," said Santa. "I've got six flat ones 2m by 2m by 1m, and three cubes 1m by 1m by 1m, so I make that 27m^3 altogether. And there's a nice space 3m by 3m by 3m on my sleigh. But I can't get them into it."

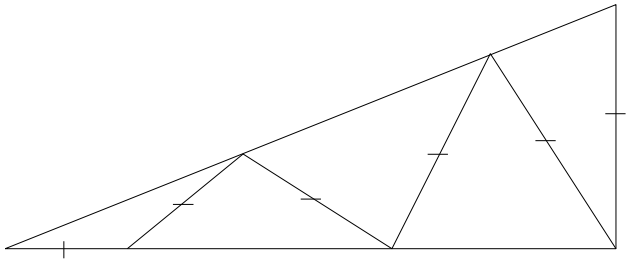
Can you help Santa?

[Hint 1: The solution is very symmetrical. Hint 2: If you can make the shapes out of building blocks or lego etc, it will help!]

Shapes Puzzle 15

The diagram on the right (which is not drawn accurately) shows a right angled triangle divided into 5 isosceles triangles, as shown in the diagram where equal sides are marked with a line.

Find the size of all the angles in the diagram.



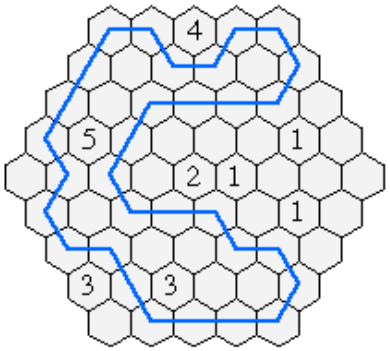
Shapes Puzzle 16

On the island of Triango, the inhabitants have come up with a novel way to avoid traffic problems. Roads between towns are not allowed to cross or branch outside of the towns. Unfortunately, however, they have not mastered the art of bridge construction, so this limits the number of roads which can be built. There are 8 small towns on the island, and as many as possible of them are connected by these direct connecting roads. There is only one road linking each pair of towns. How many of these connecting roads are there on Triango?

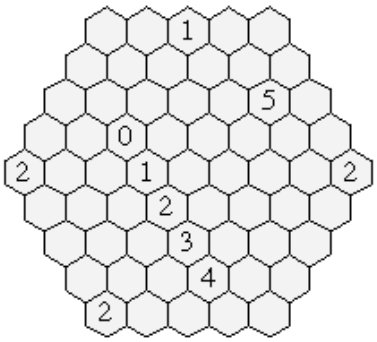
Extension 1: If there were 80 towns, how many roads would there be under the same conditions? [Hint: Don't try to draw this! You will need to find a formula for the number of roads for any number of towns.]

Extension 2: Can you **prove** that your formula must always work?

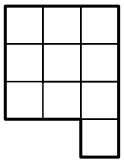
Shapes Puzzle 17



The diagram on the left shows a hexagonal grid with a closed path which travels from hexagon to hexagon, never making a **sharp** turn. Some of the hexagons through which the grid does **not** pass contain numbers showing the number of **adjacent** hexagons through which the path **does** pass. On the right is another hexagonal grid with a hidden path of the same type. Some of the hexagons through which the grid does not pass are marked with numbers in the same way as for the grid on the left. Can you reconstruct the path?

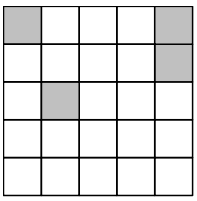


Shapes Puzzle 18



Show how to cut the shape shown on the left into three pieces which can be fitted together to make a perfect square.

Shapes Puzzle 19

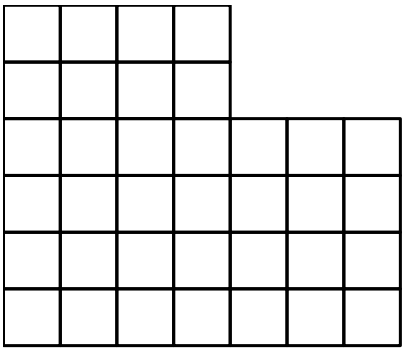


Show how to dissect the shape formed by the white squares in the grid on the left into three **congruent** shapes.

The dissection is along the grid lines – the shapes consist of whole squares.

The shaded squares are holes and do not form part of the shapes.

Shapes Puzzle 20



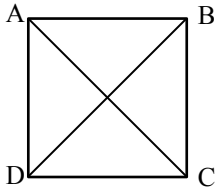
1) Show how to divide the shape on the left into two congruent shapes

2) Now show how to divide it into three congruent shapes

3) Now show how to divide it into four congruent shapes

Shapes Puzzle 21

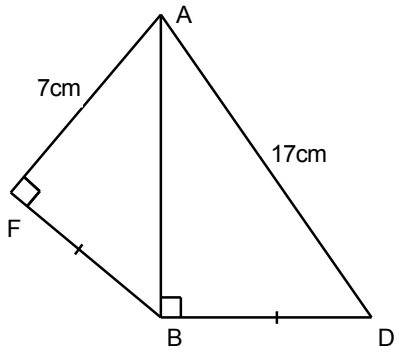
"Mark four points A, B, C and D on your paper," said the teacher, "so that the six lengths AB, AC, AD, BC, BD and CD only take two different values." "That's easy," said Angela, drawing a square as in the diagram. "See - AB, BC, CD and DA are one length, and the diagonals AC and BD are a different length. So there are just two different lengths." "My diagram's different to that - it's not square at all," said Briony. "Yes, and mine's different to both of yours," said Clare."But all three of you are correct," said the teacher. Can you draw what Briony's diagram and Clare's diagram might have looked like?



Extension: In fact every student in the class drew a different type of diagram. What is the maximum number of students there could have been in the class?

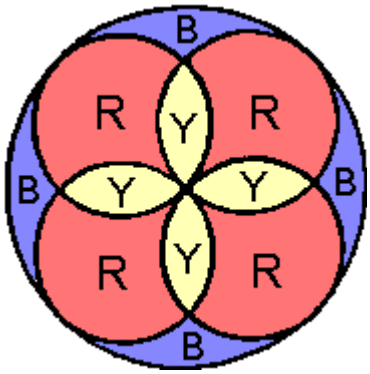
Shapes Puzzle 22

The diagram shows two right-angled triangles. BF=BD. Work out the length of AB.



Shapes Puzzle 23

The diagram shows a design for a stained glass window, which consists of a four overlapping smaller circles symmetrically placed within a larger circle. The regions are to be made from coloured glass: R = Red, B = Blue, and Y = Yellow.

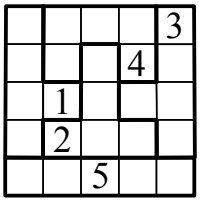


The area of each yellow section is 100cm^2. What is the area of each blue section? Explain how you work out the answer.

[Hint: The area of a circle is πr^2, but strictly speaking you don't need to know this to solve the puzzle.]

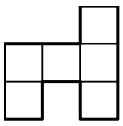
[Extension: What is the area of each red section?
You definitely do need to know $A = \pi r^2$, and also Pythagoras' Theorem, to solve this.]

Shapes Puzzle 24



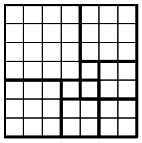
The diagram on the left shows 5 pentominoes arranged in a square. Can you place the numbers 1 to 5 in the squares of each pentomino, so that each number appears once in each pentomino, once in each horizontal row and once in each vertical line. Explain the reasoning you use.

Shapes Puzzle 25



The diagram on the left shows a hexomino – a shape formed from 6 squares. There are 35 possible hexominoes (rotations and reflections of a shape are not counted as different); how many of them could be used as the net of a cube (ie how many of them, if folded up, could form a cube)?

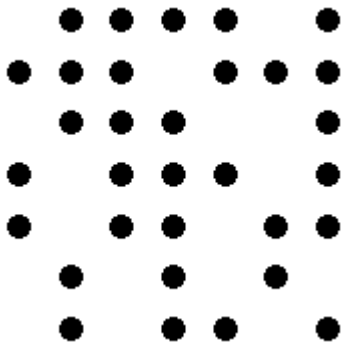
Shapes Puzzle 26



The diagram on the left shows how a 7 by 7 square can be divided into 9 smaller squares. Can you show how a 13 by 13 square can be divided into 11 smaller squares in a similar way?

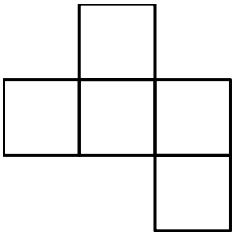
Extension: What is the least number of smaller squares which a 12 by 12 square can be divided into? What about 11 by 11? Or 10 by 10, etc? (Some cases are very obvious, others less so.)

Shapes Puzzle 27

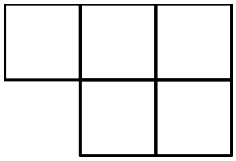


This diagram shows a 7×7 grid of equally spaced dots, but with some of the dots rubbed out. Draw 8 squares on the diagram so that each square has 4 dots at the corners, and each dot is used once, but only once, as the corner of a square. The squares can be inclined at any angle but must have right angles in the corners.

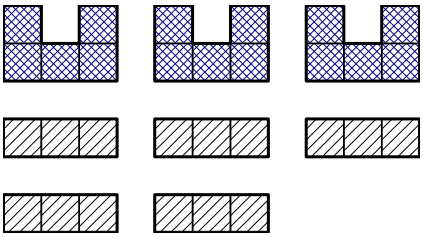
Shapes Puzzle 28



Cut out three copies of the shape on the left from squared paper, and arrange them to form a shape. Now cut out three copies of the shape on the right, and arrange them to form the **same** shape. There is only one shape which can be covered by either set of three identical pentominoes – what is it?

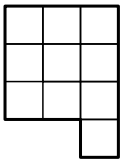


Shapes Puzzle 29



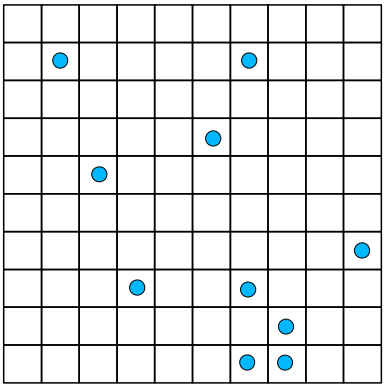
Cut out the 3 U shaped pentominoes and the 5 triominoes on the left. Then position the 3 pentominoes (with no overlapping) so that they can be covered by the 5 triominoes.

Shapes Puzzle 30

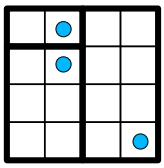


Show how to cut the shape shown on the left into three pieces which can be fitted together to make a perfect square.

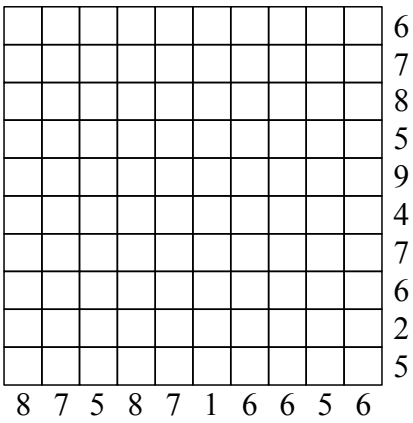
Shapes Puzzle 31



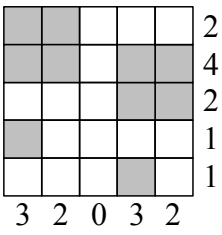
Divide the grid on the left into 10 rectangles, all of **different** sizes, so that each one has a dot in a corner square (or in an end square if the rectangle is only one unit wide. (Remember that a square is a type of rectangle, so some of the rectangles could be square.)
The example on the right shows the solution for a smaller version of the puzzle.



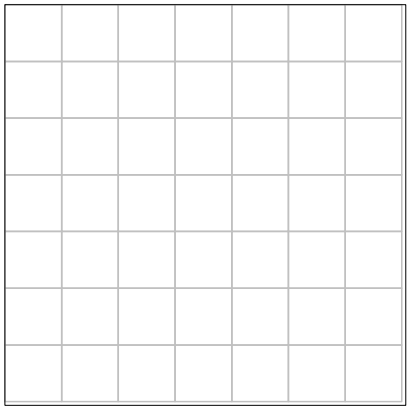
Shapes Puzzle 32



The small example on the right shows a 5 by 5 grid with several squares of various sizes shaded in. The squares cannot touch each other, not even corner-to-corner. The

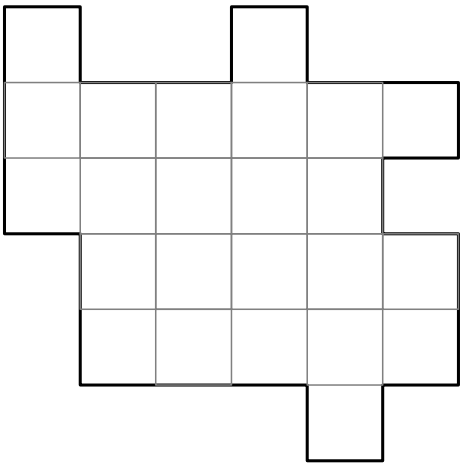


Shapes Puzzle 33



Can you show how to divide this square into 7 right-angled isosceles triangles, no two of which are the same size? All vertices of the triangles must lie on the intersections of the grid.

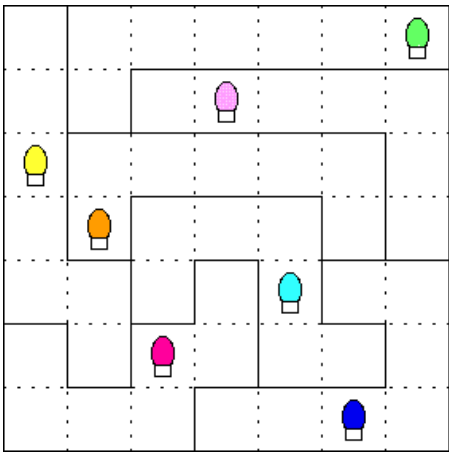
Shapes Puzzle 34



A man wishes to leave his land to be shared equally between his four children. He wants to give each child a piece of land which is the same shape as well as the same size. Unfortunately his land is a rather awkward shape, as shown on the left.

Can you show how to divide this up into four congruent pieces?

Shapes Puzzle 35

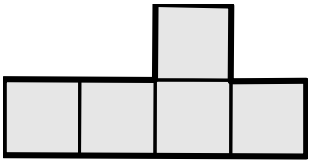


Alice has seven strings of Christmas lights which she likes to arrange in her window. Each string has seven lights, all of different colours. Last year, she worked out how to arrange them in a 7 by 7 square pattern so that there was one light of each colour in each row and each column.

She drew a diagram to remind herself how she did it, but unfortunately when she got it out this year it had become damp and faded, so she could only make out how the strings had been placed, and a few of the positions of the colours.

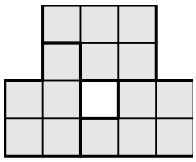
Can you help Alice? Show how she can fill in this grid with one light of each colour in each row, each column, and each "string" (or polyomino) of seven squares.

Shapes Puzzle 36

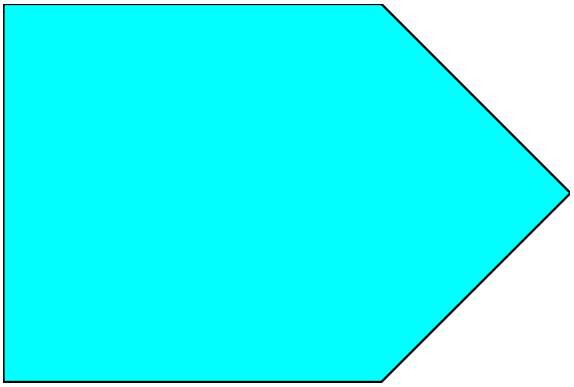


Show how to fit together five identical pentominoes, all congruent to the shape on the left, to make a shape which has a line of symmetry. Write down some indication of the logic that you used.

[As an example, the diagram on the right shows three congruent pentominoes placed together to form a shape with a line of symmetry.]



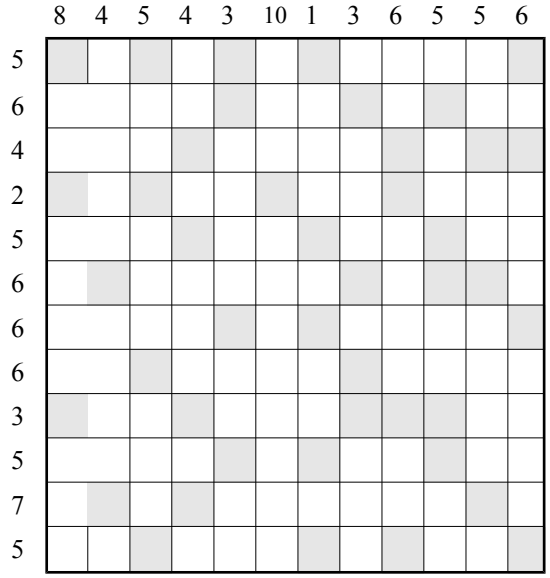
Shapes Puzzle 37



The shape on the left (which is not drawn full-size) consists of a square with sides 10cm, with a triangle with angles of 90°, 45°, and 45° attached to the right hand side so that the maximum width of the shape is 15cm. Show how to cut the shape into three pieces which can be rearranged to form a square.

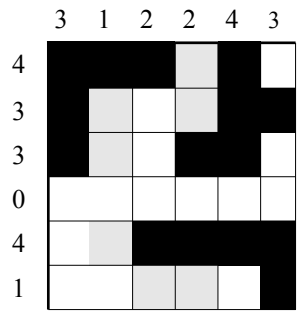
[Hint: Work out the area of the shape – this will then tell you how long the sides of the square need to be. The Theorem of an ancient Greek gentleman whose name begins with P may also be helpful.]

Shapes Puzzle 38



There are twelve different pentominoes which can be formed from 5 adjacent squares. First find all twelve of them.

The twelve pentominoes are to be placed onto the grid on the left. They must go in the white cells, not the shaded cells. The pentominoes cannot touch each other, not even diagonally at a corner.



The numbers around the edge of the grid show how many squares in that row or column form part of a pentomino.

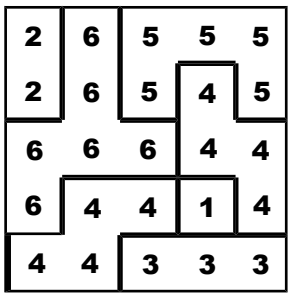
Can you place the pentominoes onto the grid so that the rules are followed?

[The example on the right shows three pentominoes placed on a small grid according to these rules.]

Shapes Puzzle 39

	4							
	4					3	2	1
	7							
			4	6	3			
			5	5	4			
			5	6	5			
							4	
4	5	6					5	
							6	

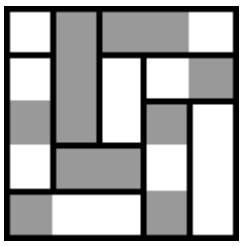
This 9 by 9 grid is filled with polyominoes (shapes formed by one or more squares). Each square in a polyomino contains an integer, which represents the number of squares in that polyomino. Polyominoes with the same number of squares cannot touch each other (except at a corner).



Can you complete the grid by filling each square with a number and marking the edges of each polyomino.

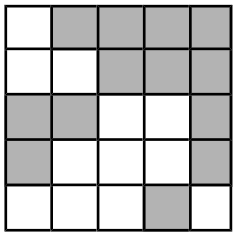
[A small example is shown completed on the right, to help explain the rules.]

Shapes Puzzle 40

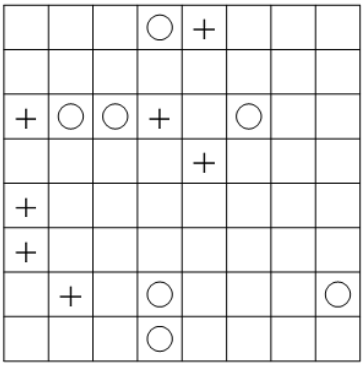


The square on the left is divided up into six 3-by-1 strips, three 2-by-1 strips and one 1-by-1 square, which are all shaded differently.

Can you divide up the square on the right in the same way?



Shapes Puzzle 41



Draw a path which passes through the centre of every square, always travelling horizontally or vertically, and forms a closed loop. The path must go straight through any square containing a cross, and turn through a right angle at the centre of any square containing a circle.

Shapes Puzzle 42

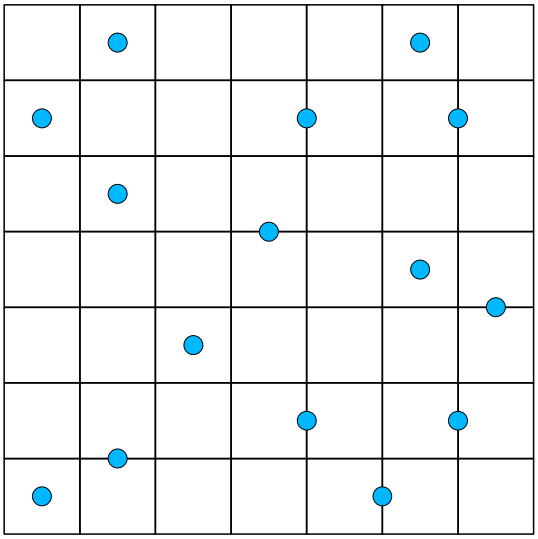
	10				14	12
		6	5			
14				4	8	
			6			6
		14			12	
10			8			
		4			14	

Divide the 7 by 7 grid into eight regions, so that each region contains two numbers, one of which is the region's area, and the other the region's perimeter. The dividing lines must goes along the lines of the grid.

Shapes Puzzle 43

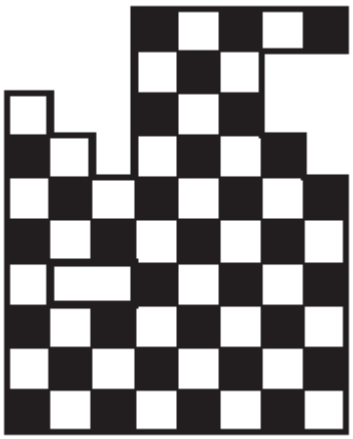
A rectangular metal box with a base 2m by 3m and a height of 2m is filled with water to a height of 40cm. Five cubes, each 1m by 1m by 1m, are lowered into the box so that they all rest on the bottom. What is the height of the water in the box now? Explain how you worked this out.

Shapes Puzzle 44



Divide the 7 by 7 grid of squares into a number of polyominoes (monominoes, dominoes, trominoes, etc...) so that each polyomino has rotational symmetry around one of the small circles.

Shapes Puzzle 45



The shape on the left has a hole in it with an area of two squares.
Can you show how to cut the whole shape, along the grid lines, into just two pieces which can be rearranged to form a perfect chessboard?

Shapes Puzzle 46

1							1
4	3			4			
				5			2
				5			2
3			5	5			
			3				
4		2	2				
1					3	4	1

Can you divide the square on the left into four **congruent** pieces such that each piece contains each of the numbers 1 to 5 exactly once. The edges of the pieces must be along the lines of the grid.

Shapes Puzzle 48

Cut out the nine polyarcs shown above, and rearrange them to form the shape shown below.