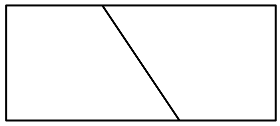
Senior Puzzle 1



A sheet of paper 10cm by 20cm is folded so that diagonally opposite corners meet.
What is the exact length of the fold (shown by a dotted line on the diagram)?
[You might want to experiment with a piece of paper first.]

Senior Puzzle 2

Daisy the cow is ruminating in her rectangular field ABCD. She is standing 19m from A, 26m from B and 34m from C. How far is she from D?
[Hint: Don't try to work out the length and breadth of the field – it is not possible!]

Senior Puzzle 3

Find the least whole number which is tripled in size when its final (right-most) digit is removed and placed in front of its first (left-most) digit.

Senior Puzzle 4

At a school in the Highlands, if it snows, the school has to close for the day. The pupils have devised a system for passing on the news quickly. Douglas, who is the deputy head's son, finds out first. He phones just two of his friends, Annie and Bernard. They then each phone just two other people, and so it goes on. One day Douglas finds out at 6.00am that the school will be closed. He phones Annie, which takes one minute. So at 6.01am two people know the news. Then he phones Bernard, which also takes a minute, and goes back to bed. So at 6.02am four people know the news (because Annie has made her first phone call too). If each phone call takes a minute, and there are 1000 pupils in the school, at what time will the whole school know the glad news?

Senior Puzzle 5

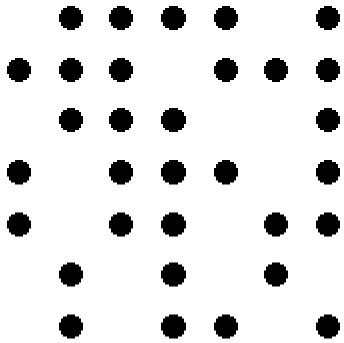
The brilliant (and eccentric) computer scientist Donald Knuth believes that every whole number can be written using just one 3, brackets, and the signs ! (which means factorial - eg 4! = 4×3×2×1=24), √ (which means square root, of course) and [ ] (which means "the integer part of" eg [7.29] = 7).
For example, 1 = [√3]   2 = [√3!]   3 = 3,   and 2116 =   (honestly - check it on a calculator)
Show how to make the number 10 in this way.
(As a bonus, see how many of the numbers from 1 to 10 you can make.)

Senior Puzzle 6

T E A S E R
S T U D Y

This TEASER is odd, so it will repay careful study. Different letters stand for different non-zero digits. Each letter on the bottom line is the sum of the two letters above it (for example U = A + S). What RESULT do you come up with?

Senior Puzzle 7



This diagram shows a 7 × 7 grid of equally spaced dots, but with some of the dots rubbed out. Draw 8 squares on the diagram so that each square has 4 dots at the corners, and each dot is used once, but only once, as the corner of a square. The squares can be inclined at any angle but must have right angles in the corners.

Senior Puzzle 8

Tony handed back the photo "So that's your two boys and Alice, eh?" he commented. "She must be a teenager now." "That's right. Their ages make a neat puzzle for you. The reciprocal of the square of her age is equal to the difference between the reciprocals of the squares of the ages of her brothers." How old is Alice?

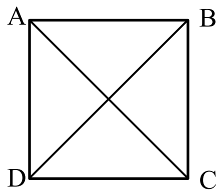
[The reciprocal of x is $\frac{1}{x}$. All three ages are whole numbers.]

Senior Puzzle 9

A right-angled triangle has a hypotenuse of 7cm and an area of 8cm^2.
Preferably without using a calculator, find the perimeter of the triangle.

Senior Puzzle 10

"Mark four points A, B, C and D on your paper," said the teacher, "so that the six lengths AB, AC, AD, BC, BD and CD only take two different values." "That's easy," said Angela, drawing a square as in the diagram. "See - AB, BC, CD and DA are one length, and the diagonals AC and BD are a different length. So there are just two different lengths." "My diagram's different to that - it's not square at all," said Briony. "Yes, and mine's different to both of yours," said Clare."But all three of you are correct," said the teacher. Can you draw what Briony's diagram and Clare's diagram might have looked like?



Extension: In fact every student in the class drew a different type of diagram. What is the maximum number of students there could have been in the class?

Senior Puzzle 11

Tweedledum and Tweedledee look alike, but Tweedledum always lies on Monday, Tuesday, and Wednesday and always tells the truth on other days, whereas Tweedledee always lies on Thursday, Friday, and Saturday and always tells the truth on other days. You come upon the two of them, and one of them (you can't tell which, so call him A) says "Today is not Sunday". The other (call him B) says "Today is not Monday."

Who is who, and what day is it?

Senior Puzzle 12

Here is a number puzzle: Take four different digits - e.g. 4 8 2 1
Arrange them to form the largest number possible: 8421
Arrange them to form the smallest number possible: 1248
Subtract these numbers: 7173
Repeat the above process: 7731 – 1377 = 6354
Repeat again…and again.

What happens eventually? You should find you get stuck at a certain "magic number". What is it?
Sometimes you get to this "magic number" in just one subtraction. How many sets of four numbers are there for which this happens?
Extension: What is the greatest number of subtractions it may take to reach the magic number?

Senior Puzzle 13

A tromino is a domino made of three squares instead of two. If we colour each square of a tromino either red, white or blue, there are 18 different trominos which can be formed. Three of them are shown below.

R	R	B

R	W	B

R	W	R

Make a list of the 18 possible trominos. A set of these have been placed in a rectangle below, but the dividing lines have been erased. Can you reconstruct the eighteen trominos?
[Hint: There is no need for trial and error. There is one tromino which must be in a certain place, and fixing this one then opens up more information about some of the other trominos, and so on until the puzzle is complete.]

R	R	W	B	R	W	B	W	B
W	B	W	B	W	W	W	B	R
B	W	R	R	R	W	W	W	W
R	B	W	R	B	B	R	R	R
W	W	W	R	R	B	R	B	B
B	B	W	R	R	B	B	B	R

Senior Puzzle 14

N is a four digit positive integer which does not end in a zero, and R(N) is the four-digit number obtained by reversing the digits of N.
(For example, R(3275) = 5723) Find all such integers N from which R(N) = 4N + 3
[There is no need for any trial and error here – it is possible to tackle this logically. Hint: the two digit number "AB" is written algebraically as 10A + B.]

Senior Puzzle 15

ABCD is any convex quadrilateral. P, Q, R and S are the midpoints of the sides AB, BC, CD and DA. Prove that the area of ABCD is twice the area of PQRS.

Senior Puzzle 16

Find all pairs of positive integers m and n, where n is odd, such that $\frac{1}{m}+\frac{4}{n}=\frac{1}{12}$.

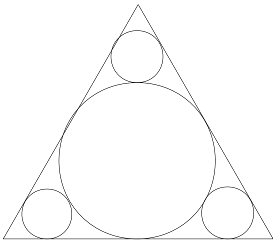
Senior Puzzle 17

To win at noughts and crosses, no matter what the size of the board, you have to get *three* of the same symbol in a line. On a normal 3×3 noughts and crosses grid there are 8 possible winning lines: 3 horizontal, 3 vertical and 2 diagonal lines.
(i) If you play noughts and crosses on a 4×4 grid (but you still win by getting three in a line), how many winning lines are there?
(ii) What if you play on an n×n grid?
(iii) What if you play on (or in!) an n×n×n cube?
[A prize if you answer the first two parts of this; a bonus if you do the third part as well!]

Senior Puzzle 18

Find three whole numbers such that the sum of any pair of them is a perfect square. [eg 4, 12 and 21 nearly work because 4+12=16, 4+21=25, but 12+21=33 which is not a square number.]   Is it possible to do this for any three perfect squares? If not, what has to be true about the squares?

Senior Puzzle 19



The diagram shows four touching circles, each of which also touches the sides of an equilateral triangle with sides of length 3. What is the area of the shaded region? [Leave your answer in terms of π.]

Senior Puzzle 20

A class consists of 6 pairs of twins. Their teacher wishes to divide them into teams for a quiz, but wants to avoid putting any pair of twins into the same team.

- In how many ways can she split them into two teams of six?
- In how many ways can she split them into three teams of four?

Senior Puzzle 21

How many nine digit numbers are there which satisfy the following conditions:
Each of the digits 1 to 9 appears once in the number.
The numbers 1 to 5 appear in the correct order within the number.
The numbers 1 to 6 do not appear in the correct order within the number.
[An example of such a number is 916238457.]

Senior Puzzle 22

Prove that $\cos 11° + \cos 83° + \cos 155° + \cos 227° + \cos 299° = 0$

Senior Puzzle 23

S is a subset of the set of numbers {1, 2, 3, 4, ..., 2008} which consists of 756 distinct numbers. Show that there are two distinct numbers *x* and *y* in S such that *x*+*y* is divisible by 8.

Senior Puzzle 24

If *x*, *y* and *y* are positive integers, find all solutions of the simultaneous equations:

$$x + y - z = 12$$
$$x^2 + y^2 - z^2 = 12$$

Senior Puzzle 25

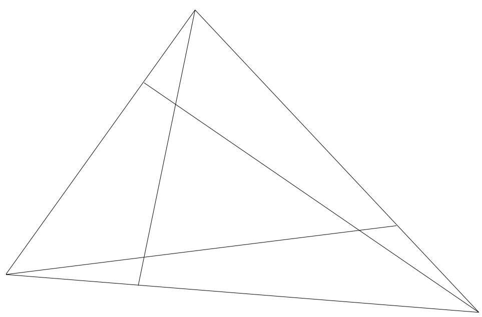
Santa has five workshops scattered around the globe in hidden places, codenamed A, B, C, D and E. He needs to connect them with a regular sleigh service for transporting elves and toys. But due to the recession he wants to run as few services as possible. (He doesn't need to run a direct service connecting each workshop. For example, if there is service from A to B and one from B to C, there doesn't need to be a service from A to C. As long as there is some way for elves to get from one workshop to another, that is satisfactory.) In how many different ways can he choose which services to run?

[Extension: There is a neat formula for the number of different ways to connect n workshops, which you may be able to **spot**, but the **proof** is too difficult for a BMO question.]

Senior Puzzle 26

The diagram shows a large triangle divided up by three straight lines into four smaller triangles and three quadrilaterals. The sum of the perimeters of the three quadrilaterals is 25cm. The sum of the perimeters of the four smaller triangles is 20cm. The perimeter of the large triangle is 19cm. What is the sum of the lengths of the three straight lines which divide up the large triangle?

[You need to explain the method you use to work this out.]



Senior Puzzle 27

Given that 34! = 295232799cd96041408476186096435ab000000, work out the missing digits a, b, c and d.

Senior Puzzle 28

Let x, y and z be positive numbers such that $x^2+y^2+z^2=1$. Prove that:

$$x^2yz+xy^2z+xyz^2\leq\frac{1}{3}$$

Senior Puzzle 29

A set of positive integers is defined to be "wicked" if it does not contain three consecutive integers. (Therefore the empty set is counted as wicked.) Find the number of wicked subsets of {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}.

Senior Puzzle 30

Mrs Smith was visiting her old friend Mrs Jones, whom she hadn't seen since they were at school together. "So you have three children now," said Mrs Jones. "What are their ages?" "Since I know you like puzzles," said Mrs Smith, "I'll give you a couple of clues. The sum of their ages is 13, and the product of their ages is the number of your house." After trying to solve the puzzle for a couple of minutes, Mrs Jones said (correctly), "Hang on! You haven't given me enough information."

But you *do* have enough information to work out the number of Mrs Jones' house. What is it? (And what can you say about Mrs Jones' children?) Explain how you work this out.

Senior Puzzle 31

How many five-digit even numbers are there with all the digits different?
[Hint: You need to find and explain a method for working this out.]

Senior Puzzle 32

Three parts to this puzzle:

- How many zeros are there at the end of 100! ?
- Explain why there is no number n such that n! ends in 5 zeros.
- List all the numbers z (where $z \leq 30$) such that there is no number n! ending in z zeros.

[Note: You do NOT need to work out 100! to answer this. But you do need to explain your answers.]

Senior Puzzle 33

Each of the ten letters which occurs in the seasonal message "A MERRY XMAS TO ALL" represents a different digit from 0 to 9. Each word in the message is a square number, and also the sum of the digits in each word is a square number. Can you work out what each digit represents? Explain how you did this.