Numbers Puzzle 1

Arrange the digits 1 to 9 in a row so that each pair of consecutive digits forms a two-digit number that is the product of two single-digit numbers.
For example, if you started 426..., the 42 would be allowed because $42 = 6 \times 7$, but 26 would not be allowed because it is 2×13, but 13 is not a single digit.
The solution can be obtained logically, without any trial and error. Try to explain how you find your solution.

Numbers Puzzle 2

Mrs Smith was visiting her old friend Mrs Jones, whom she hadn't seen since they were at school together. "So you have three children now," said Mrs Jones. "What are their ages?" "Since I know you like puzzles," said Mrs Smith, "I'll give you a couple of clues. The sum of their ages is 13, and the product of their ages is the number of your house." After trying to solve the puzzle for a couple of minutes, Mrs Jones said (correctly), "Hang on! You haven't given me enough information."

But you do have enough information to work out the number of Mrs Jones' house. What is it? (And what can you say about Mrs Jones' children?) Try to explain how you find your solution.

Numbers Puzzle 3

At a recent school fete, exactly 500 people attended and exactly £500 was raised. The prices were as follows:
Parents: £2 per person Other adults: £3 per person Children £0.48 each
How many of each category attended the fete?
There are two possible answers – can you find them both?

Numbers Puzzle 4

```
FORTY
  TEN
+ TEN
SIXTY
```

The addition sum on the left is certainly correct. But can you replace each letter by a different digit (using the same digit every time the letter occurs) so that the sum is still correct? Try to explain the order in which you work out the letters.

Numbers Puzzle 5

"Twenty Four" is a card game played by four players with a pack of 36 cards – the Ace (1) to 9 of each suit. The cards are shuffled and dealt out and each player has a pile of 9 cards face down in front of her. The players then simultaneously turn over their top cards to reveal four numbers. The first player to write down a way of making 24 using only + , – × and ÷ signs and brackets (no powers, no roots, no factorials...) shouts "Twenty-four" and claims the four cards which are added to the bottom of her pile. For example, if the cards were 1,2,5 and 9, you could write $(9 – 1) \times (5 – 2)$ to win the round.
A new round then starts. When a player runs out of cards, the game ends, and the player with the most cards is the winner.

Two puzzles: How can you make 24 using 1,4,6,7? How can you make 24 using 3,3,7,7?

[Other puzzles could be 4,4,7,7 or 5,5,5,1 or 3,3,8,8]

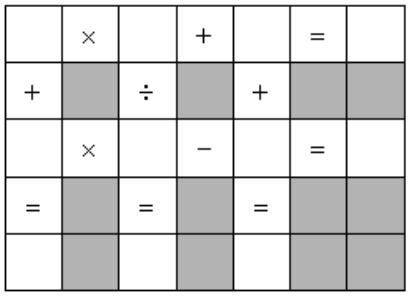
Numbers Puzzle 6

Find a nine digit number which is formed from the digits 1 to 9 used once each, with the property that the number formed by the first n digits is divisible by n. Explain how you found your answer.
[For example, 3214 nearly works for a 4 digit version of this, because 3 is divisible by 1, 32 is divisible by 2, 321 is divisible by 3, but 3214 is not divisible by 4.]

Numbers Puzzle 7

A certain country has three kinds of coins, each worth a different integer number of dollars. Alice has four coins in her pocket, worth a total of 28 dollars; Bob has five coins worth a total of 21 dollars. Each has at least one coin of each kind. Find the value of each of the three kinds of coin. Try to explain the logic you use.

Numbers Puzzle 8



Fit the numbers from 1 to 11 once each into the spaces in this cross number puzzle, so that each sum across and down is correct. Try to explain your logic as far as possible.

Numbers Puzzle 9

I am thinking of a three-digit number which contains three different digits. The following are all perfect squares:

- The sum of the first digit and the number formed by the second and third digits;
- The product of the first digit and the number formed by the second and third digits
- The sum of the three digits.

What number am I thinking of? Explain how you found the number.

Numbers Puzzle 10

Here are three groups of numbers:
Group 1: 15, 19, 24 Group 2: 11, 30, 36 Group 3: 20, 22, 36
In each group, the numbers can be combined using +, – , × ÷ and brackets to give the **same** answer:
Group 1: $24 \div (19 - 15) = 6$ Group 2: $(30 + 36) \div 11 = 6$ Group 3: $20 + 22 - 36 = 6$

Can you do the same with the following three groups:
Group 1: 3, 15, 18 Group 2: 10, 13, 36 Group 3: 24, 27, 39

Numbers Puzzle 11

Can you find a whole number n greater than 1 such that the product of all its factors (including itself) is n^3?

Extension: Can you give a formula for numbers of this type?

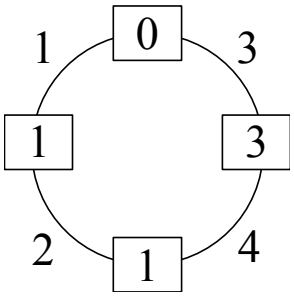
Extra extension: There are two such formulae. Can you find them both?

Numbers Puzzle 12

One of Santa's elves has been out delivering chocolate snowmen to hang on trees at Santa's various grottos. "How did you get on?" asked Santa on his return.
"It was odd, really," replied the elf. "I visited seven grottos. At the first grotto they wanted exactly half my snowmen, plus half a snowman." "Half a snowman!" growled Santa, who was finding being jolly all the time a bit wearing, "I hope you haven't been chopping up my best decorations!"
"Let me finish," said the elf. The odd thing was, at each of the seven grottos the same thing happened – they took half of the snowmen I had, plus half a snowman. At the end, I had no snowmen left. And I didn't chop up a snowman at all."
How many snowmen did the elf deliver?

Bonus. If the same thing happened when visiting n grottos, how many snowmen would the elf deliver?

Numbers Puzzle 13

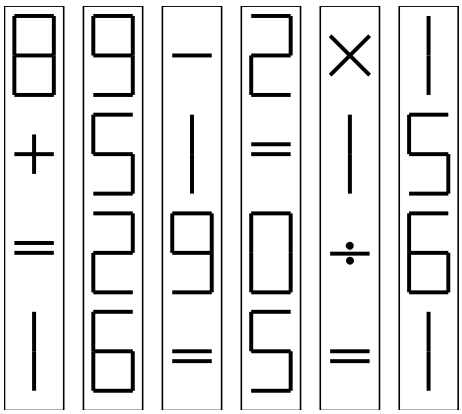


The diagram shows a ring of four boxes, with an integer (whole number) placed in each box. The numbers in each pair of adjacent boxes are added and the totals written on the connecting lines.
The totals are found to be 1, 2, 3 and 4 (though not in that order).

Prove that it is not possible to do this with five boxes – ie to place an integer in each box so that the totals of adjacent boxes are 1, 2, 3, 4 and 5 in some order.

Extension 1: What is the next number after 4 for which this is possible? Show a possible arrangement for this number of boxes.
Extension 2: For which numbers in general is it possible to do this?

Numbers Puzzle 14



On the left are six strips each containing four numbers. Can you cut out the strips and arrange them so that reading across there are four true equations? (Note that the strips can be turned upside down.) Explain the reasoning that you use to work this out.

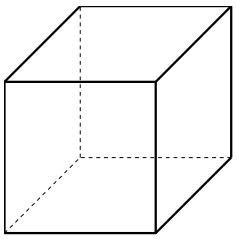
Numbers Puzzle 15

A man wishes to sell a puppy for £11. A customer wants to buy it but only has foreign currency, which has coins of three types: round, square and triangular. The exchange rate for the foreign currency is:

11 round coins = £15; 11 square coins = £16; 11 triangular coins = £17.

Is it possible for the customer to pay exactly the right amount for the puppy? If so, how many of each type of coin should the customer pay? Explain how you work this out.

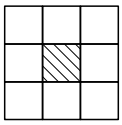
Numbers Puzzle 16



On the left is a cube, which has 12 edges and 8 vertices. Ann wants to label the twelve edges with the numbers 1 to 12 in such a way that the three edges meeting at each vertex add up to the same sum.

1) Prove that it is impossible to do this.
2) Ann now decides to replace one of the numbers 1 to 12 with the number 13, and make the sum of the three edges meeting at each vertex come to 21. Which number must she replace with 13? Show how to label the edges so that the total ay each vertex is 21.

Numbers Puzzle 17



Place the numbers 1 to 8 in the unshaded squares of this grid so that on each of the four sides the middle number is the sum of the two corner numbers. Explain the reasoning you use.

Numbers Puzzle 18

In a (normal) magic square, each row, column and diagonal has the same sum. For example, here is a 3 by 3 magic square, containing 9 distinct positive integers:

2 9 4
7 5 3
6 1 8

Can you find a 3 by 3 *multiplication* magic square using 9 distinct positive integers where each row, column and diagonal has the same *product*?

Extension: Try to find the magic square which has the *smallest* product.

Numbers Puzzle 19

36	6	25	10	13
55	49	30	89	4
15	144	45	12	7
9	91	2	81	3
1	11	5	17	21

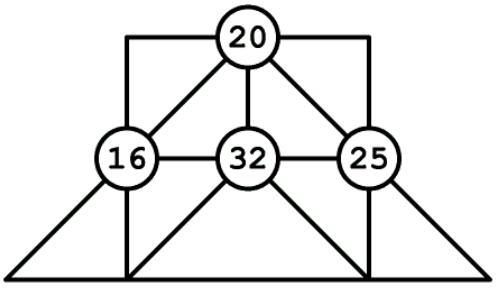
The grid on the right contains six sets, each containing 4 numbers:

- a set of 4 square numbers
- a set of 4 triangular numbers
- a set of 4 prime numbers
- a set of 4 Fibonacci numbers
- a set of 4 odd numbers
- a set of 4 factors of 60

No two members of a set are in the same row or column – for example 36 and 25 cannot both be in the set of four square numbers.
Can you work out which numbers are in each of the six sets, and therefore which number is left over at the end ie is not in any of the sets?

Numbers Puzzle 20

Place a different number from 1 to 9 inside each of the nine triangles so that each of the four circled numbers is the sum of the numbers in the triangles that touch it.



Numbers Puzzle 21

$$ABCBA = D \times BE \times BFFA$$

In this cryptarithm, each letter represents a different digit, but a letter represents the same digit every time it occurs.
The 5 digit palindromic number ABCBA has been written as the product of its **prime** factors D, BE and BFFA.
Can you work out the numbers involved? Explain the reasoning you use.

Numbers Puzzle 22

Alice and Betty went to the corner shop and bought two boxes of sweets to give to their boyfriends on Valentine's day. "£4.90 please," said the shopkeeper, and Alice paid the money. Outside the shop, Betty said "Didn't you notice – he multiplied the amounts on his calculator instead of adding them." "It doesn't matter," said Alice, "we paid the right amount anyway." What were the prices of the two boxes of sweets?

Numbers Puzzle 23

Four people, A, B, C and D each have two coins in their pocket. A has twice as much money as B, who has twice as much as C, who has twice as much as D. What coins does each have?
(The possible coins are 1p, 2p, 5p, 10p, 20p, 50p, 100p and 200p.)

Numbers Puzzle 24

```
  DE
 ×ED
 ---
 DDE
 DE0
 ---
DEDE
```

In the long multiplication sum above, each "D" stands for an odd digit, and each "E" for an even digit. The "0" is a zero, put there because on that line we are multiplying by the tens digit. Can you work out what the sum is? Explain the logic that you use.

Numbers Puzzle 25

	Square Number	Prime Number	Multiple of 3	Factor of 280
Factor of 120				
Greater than 12		*		
Even				
Odd				

Fit the numbers from 1 to 16 into the table above so that each number is used once, and all the numbers in each row and column are as described, for example the number in the box marked with a star must be greater than 12, and a prime number.

Numbers Puzzle 26

In the equations below, the eight letters *a* to *h* each represent a different single digit number between 1 and 9.

$$a^b \times b^a = c(c-1)$$
$$d^e \times e^d = a(a-1)$$
$$f = g^2 - a^2$$
$$h^b = g^a$$

Which number between 1 and 9 is **not** used? [You will need to work out what number each of the values *a* to *h* represent.]

Numbers Puzzle 27

```
    A
  MERRY
+  XMAS
 TURKEY
```

In the addition sum on the left, each letter stands for a different digit (so if A is 5, M cannot also be 5). A letter stands for the same number everywhere it occurs (so if the M in "MERRY" is 5, then the M in "XMAS" must also be 5. Can you work out what each letter stands for?

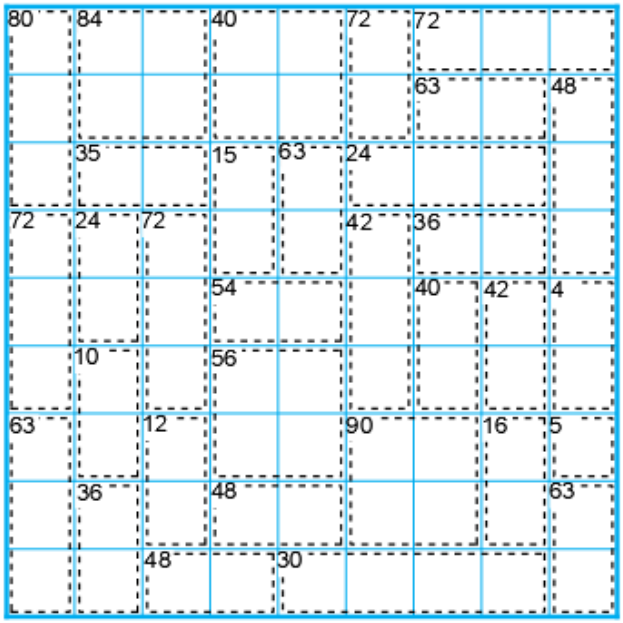
Numbers Puzzle 28

Welcome to 2006!
Can you insert mathematical symbols + – × ( ) ! into the numbers below so that the resulting expression is equal to 2006?

8 7 6 5 4 3 2 1

Each symbol may be used as often as required. The digits must remain in the order given.
[Hint: 3! Is called "3 factorial" and means 3 × 2 × 1, so equals 6. Similarly, 4! = 4 × 3 × 2 × 1 = 24, etc]

Numbers Puzzle 29



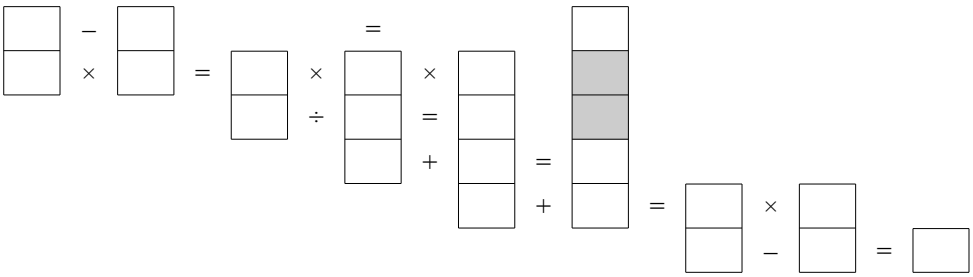
This is a "product sudoku". It has to be filled with the numbers 1 to 9, so that:

- Each row contains the digits 1 to 9
- Each column contains the digits 1 to 9
- The *product* of the digits in each region marked with a dotted line is equal to the number in the top left hand corner of the region

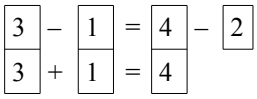
[For those familiar with normal sudoku, note there are no 3 by 3 squares on this grid – that rule does not apply.]

Solve the puzzle, and show (partly) how you did it by marking in some way (eg using colours) the first ten numbers you worked out.

Numbers Puzzle 30



The above arrangement shows six calculations. Fill in the numbers 1 to 9 in the unshaded boxes, so that all the calculations are correct. All the numbers in a vertical column are identical. Each number from 1 to 9 is used once for a column.
[A small example just using the numbers 1 to 4 is shown on the right.]



Numbers Puzzle 31

Here are some musical sums. Each letter represents a different digit from 0 to 9. Each letter represents the same dfigit every time it occurs. Can you replace the letters with the correct digits?

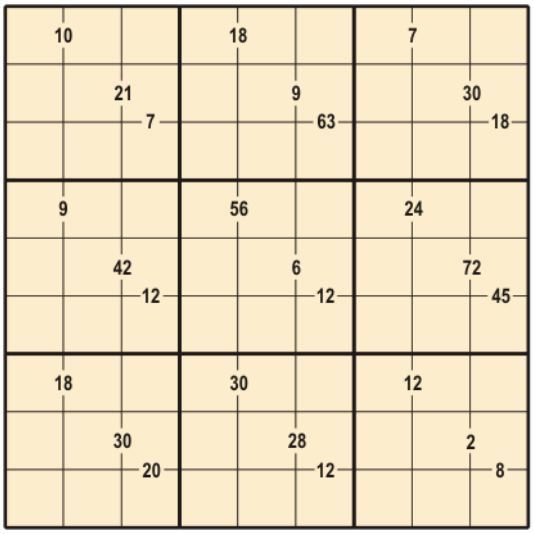
```
RE + MI = FA
DO + SI = MI
LA + SI = SOL
```

Numbers Puzzle 32

			8
			15
			22
11	13	21	

Place the numbers from 1 to 9 into the table above so that each number is used once, and the numbers in each row and column add up to the total given. Give some explanation how you solved the puzzle.

Numbers Puzzle 33

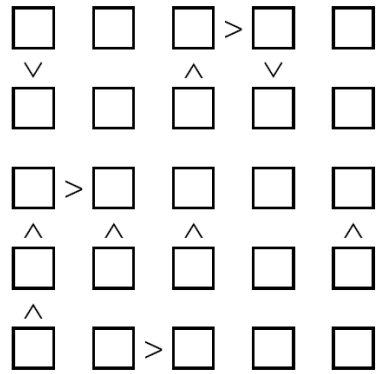


This is a sudoku puzzle with a difference. Some of the lines between two cells have a number on them; this is the product of the numbers in these two cells.

The normal rules of sudoku also apply (ie the numbers 1 to 9 must appear once each in each row, and in each column, and in each square block).

Can you solve this puzzle. Give some explanation of how you set about it.

Numbers Puzzle 34



Place the numbers 1 to 5 in this diagram, so that each number appears once in each row, and once in each column. Also, The "greater than" signs must be true – for example, the number in the top left hand corner must be greater than the number immediately below it.

Try to explain your reasoning, at least to show how you found the first couple of numbers.

Numbers Puzzle 35

Arrange the digits 1 to 9 in a row so that the number formed by any two consecutive digits is the product of two single digit numbers.
Explain the logic behind the way you worked this out.

[For example, if we had just the digits 1 to 4, then 3214 would be a solution, because $32 = 4 \times 8$, $21 = 3 \times 7$ and $14 = 2 \times 7$. But 3412 would not be a solution, because $34 = 2 \times 17$ or 1×34, and 17 and 34 are not single digit numbers.]

Numbers Puzzle 36

The number 153 can be made using just the operators +–×÷, powers and brackets, and its own digits: $153 = 51 \times 3$
So can 127: $127 = 2^7 - 1$. Can you show how to make 289, 688, 736, and 1285 in the same way?

Numbers Puzzle 37

A group of 100 monkeys have been hoarding bananas. Each monkey has at least one banana, but some monkeys have quite a few. Between them they have a total of 1716 bananas. Prove that there must be at least 4 monkeys who have the same number of bananas.

Numbers Puzzle 38

Using just the four arithmetic operations, along with brackets, can you form the number 24 from each of the following sets of four numbers. (No powers, roots, factorials, etc are allowed.)

1569 2469 4778 3377

(As an example, if we had the set of numbers 2448, we could make 24 by $(8 - (4 \div 2)) \times 4$ or by $(8 \times 4) - (2 \times 4)$ etc)

Numbers Puzzle 39

Each person in a school has a security number to open their locker, which consists of the digits 1 to 9 with each digit occurring exactly once. Jane notices that for her number:

- the sum of the digits 1 and 2 and all the digits in between is 12
- the sum of the digits 2 and 3 and all the digits in between is 23
- the sum of the digits 3 and 4 and all the digits in between is 34
- the sum of the digits 4 and 5 and all the digits in between is 45

Jane's friend Emma finds that the same is also true for her number, which is bigger than Jane's.
What is Jane's number? Explain how you worked this out.

Numbers Puzzle 40

Four teams (City, Rovers, United and Wanderers) play a football tournament. Each team plays each of the other teams once. Two points are awarded for a win and 1 for a draw.

United score 5 points, Rovers 3 points and Wanderers 1 point.
Thirteen goals are scored in all, seven of them by Rovers. City score no goals at all.
Rovers beat Wanderers by 4 goals to 1.

What was the score in the match between Wanderers and United? Explain how you worked this out.

Numbers Puzzle 41

Each of the ten letters which occurs in the seasonal message "A MERRY XMAS TO ALL" represents a different digit from 0 to 9. Each word in the message is a square number, and also the sum of the digits in each word is a square number. Can you work out what each digit represents? Explain how you did this

Numbers Puzzle 42

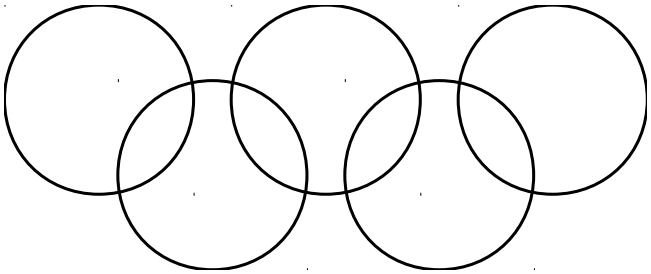
A certain five digit number (with no repeated digits) is reversed if you multiply it by 4. So $ABCDE \times 4 = EDCBA$.

Find this number, and explain the method that you used.

Numbers Puzzle 43

The teacher wrote a large whole number on the blackboard, and said "I am going round the class and I want each of you to tell me a factor of this number." Johnny was asked first, so he decided to play safe. "1 is a factor," he said. Jane said "2 is a factor." And so it went on round the 30 members of the class, until Zena said "30 is a factor." The teacher made no comment until they had all answered, then said, "As it happens, only two of you gave wrong answers, and those two answered one after another." Which two numbers were not factors of the large number?

Numbers Puzzle 44



Show how to fit the numbers 1 to 9 into the nine spaces inside the rings in the diagram on the left, so that the total of the numbers in each ring is 11.

Numbers Puzzle 45

A, B C, D , E, and F are six positive integers (whole numbers). Here are five equations involving these numbers:

$$B * (A+C+D+E+F) = 225$$

$$C * (A+B+D+E+F) = 301$$

$$D * (A+B+C+E+F) = 369$$

$$E * (A+B+C+D+F) = 400$$

$$F * (A+B+C+D+E) = 525$$

Find the values of these six numbers. Explain how you work out the answer.

Numbers Puzzle 46

At a school in the Highlands, if it snows, the school has to close for the day. The pupils have devised a system for passing on the news quickly. Douglas, who is the deputy head's son, finds out first. He phones just two of his friends, Annie and Bernard. They then each phone just two other people, and so it goes on. One day Douglas finds out at 6.00am that the school will be closed. He phones Annie, which takes one minute. So at 6.01am two people know the news. Then he phones Bernard, which also takes a minute, and goes back to bed. So at 6.02am four people know the news (because Annie has made her first phone call too). If each phone call takes a minute, and there are 1000 pupils in the school, at what time will the whole school know the news?

Explain how you worked this out.

[Hint: You need to build a sequence and work out the rule for finding the next term.]

Numbers Puzzle 47

Four three-digit numbers are in arithmetic progression, and each successive number has one more factor than the number before it.
What are the four numbers?

[Hints: "In arithmetic progression" simply means that the gap between successive numbers is equal – for example
7, 15, 23, 31, 39 are in arithmetic progression since the gap between successive numbers is 8.

What type of number has an odd number of factors?

What is the rule for working out the number of factors of a number? (It is based on the prime factorisation of the number.)]

Numbers Puzzle 48

Alice and Betty went to the corner shop and bought two boxes of sweets to give to their boyfriends on Valentine's day. "£4.90 please," said the shopkeeper, and Alice paid the money. Outside the shop, Betty said "Didn't you notice – he multiplied the amounts on his calculator instead of adding them." "It doesn't matter," said Alice, "we paid the right amount anyway." What were the prices of the two boxes of sweets?

Numbers Puzzle 49

A few years ago the theory was put forward that Real Madrid's top players played in shirts bearing prime numbers (Beckham: 23, Zidane: 5, Ronaldo: 11 etc). So our local club, No-Hopers United, decided that all their 11 players should wear prime numbers, to boost their performance. So each player was given a shirt with a prime number less than 45. What's more, since football is a team game, they made sure that the mean of the players' numbers was also a prime number different from the number of any of the players.

What were the 11 numbers used?

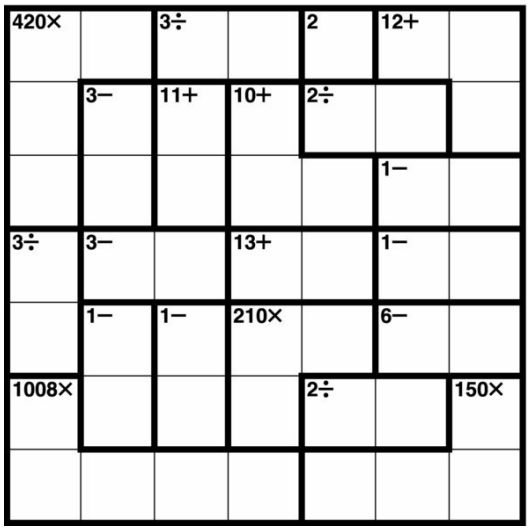
Numbers Puzzle 50

There is a cheese shortage, and everyone is stocking up. By the time Ann and Belinda get to the shop, there are only 10 pre-packed lumps of cheese left, with weights of 15, 16, 18, 19, 23, 24, 25, 27, 28, 31 and 37 ounces. Between them Ann and Belinda buy up all the remaining cheese, except for one lump which looks a bit mouldy. When they get home, they find that the total weight of Belinda's cheese is exactly twice that of Ann's cheese. What was the weight of the mouldy lump?
[Hint: There is no need for any trial and error here - you do **not** need to work out who bought which pieces of cheese.]

Numbers Puzzle 51

This is a "KenKen" puzzle.

Each row and column must contain all the numbers from 1 to 7.
Also, the numbers within each section outlined in bold (called a "cage") must combine using the operation indicated to give the answer in the top left hand corner of the cage.



Numbers Puzzle 52