# Intermediate Puzzle 1

Sarah was out walking in the countryside when she came to a wooden bridge across a stream. As she approached it, out popped a troll. "Halt" he said, "I am the Double Crossing Troll. If you cross this bridge, I will double the money you have in your pocket. But then you must pay me my fee."

"How much?" asked Sarah, and the troll told her. Sarah checked the money in her pocket. "OK" she said, and marched bravely across the bridge. When she got to the other side, she felt in her pocket, and sure enough her money had doubled. She threw the troll his fee. She was about to go on her way, when a thought occurred to her. She walked back across the bridge, and checked her pocket. Yes, her money had doubled again. So she threw the troll his fee, and crossed for a third time. Again her money doubled, and after paying the troll again, she counted it up. "Right," she thought, "I've now got exactly twice as much as I started with. So if I just keep crossing the bridge all afternoon…."

But then she remembered all the fairy tales she had read about what happened to people who were too greedy. "All right," she thought "I'll just cross 3 more times, and then go home." And that is what she did – each time, her money doubled, and then she paid the troll. When she got home, she counted her money, and found she had … **how many times what she had set off with**?

Intermediate Puzzle 2
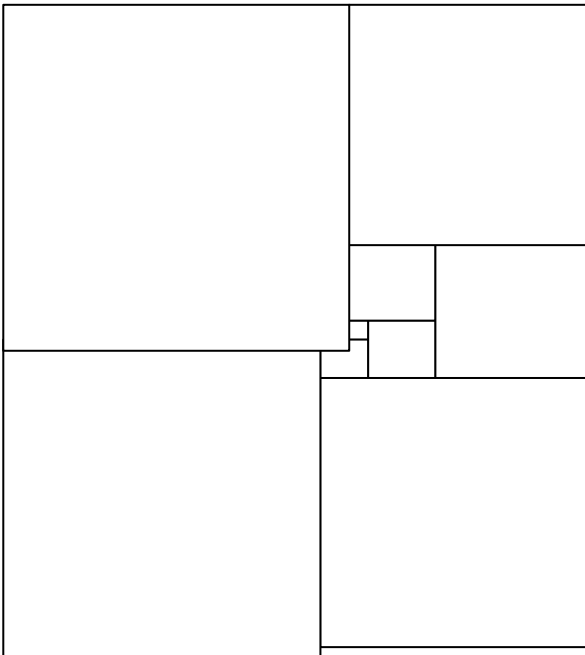
This diagram shows a rectangle divided up into a number of squares of different sizes. The smallest square has sides x cm, and the next smallest has sides y cm long. If the sides of the rectangle are the smallest possible whole numbers, what must the values of x and y be and what is the size of the rectangle?

[This puzzle illustrates part of the method used by a group of mathematicians who discovered in the 1930's a method of "squaring the square" – dividing up a **square** into a number of smaller squares, all of different sizes, with lengths that are whole numbers. The smallest known such square is made up of 24 smaller squares.]

Intermediate Puzzle 3

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        A
    MERRY
  + XMAS
   TURKEY
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In the addition sum on the left, each letter stands for a different digit (so if A is 5, M cannot also be 5). A letter stands for the same number everywhere it occurs (so if the M in "MERRY" is 5, then the M in "XMAS" must also be 5. Can you work out what each letter stands for?

Intermediate Puzzle 4

Alan, Betty and Colin were swapping marbles in the playground. First, Betty gave Alan as many marbles as he already had. Then, Colin gave Betty as many marbles as she had left after giving some to Alan. Then, Alan gave Colin as many marbles as Colin had left after giving some to Betty. After all this, they discovered that they each had the same number of marbles. If they started with 240 marbles between them, how many did each one start with?

Intermediate Puzzle 5

There is a cheese shortage, and everyone is stocking up. By the time Ann and Belinda get to the shop, there are only 10 pre-packed lumps of cheese left, with weights of 15, 16, 18, 19, 23, 24, 25, 27, 28, 31 and 37 ounces. Between them Ann and Belinda buy up all the remaining cheese, except for one lump which looks a bit mouldy. When they get home, they find that the total weight of Belinda's cheese is exactly twice that of Ann's cheese. What was the weight of the mouldy lump?
[Hint: There is no need for any trial and error here - you do not need to work out who bought which pieces of cheese.]

Intermediate Puzzle 6

I am thinking of a five digit number.
The second digit is twice the first.
The third digit is the sum of the first and last.
The fourth digit is the sum of the first and second.
The last digit is a quarter of the sum of all the digits.
What is the number?

Intermediate Puzzle 7

Susan bought three items and took them to the assistant, who worked out the cost on a calculator. "£5.70 please," he said. But Susan was watching him, and suddenly said "Hang on - you've multiplied the three amounts instead of adding them!" The assistant seemed unconcerned. "You prefer me to add them?" he said, and did so. "£5.70 please," he repeated. "But…" said Susan. "Check it yourself, if you like," said the assistant. She did so, and handed over the money. What were the three prices?

Intermediate Puzzle 8
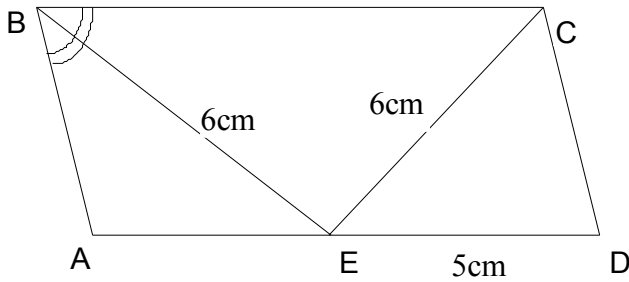
In this parallelogram, the line BE bisects angle ABC. How long is AB?

Intermediate Puzzle 9

The diagram shows two right-angled triangles. BF=BD. Work out the length of AB.

Intermediate

The social club had a passport photo displayed in a rectangular array of the rows were getting too long, they two more rows," said one member, row." "Yes,", said another, "but if we in each row." How many members

Extension: How many rectangular Which arrangement is closest to a
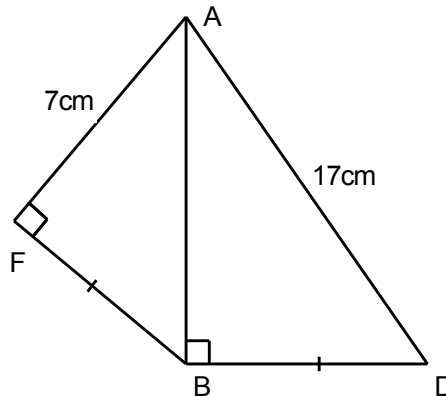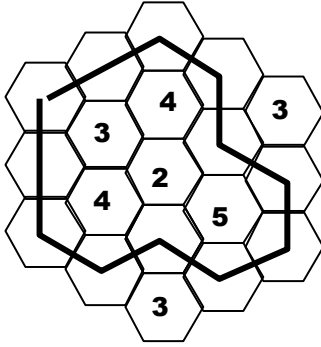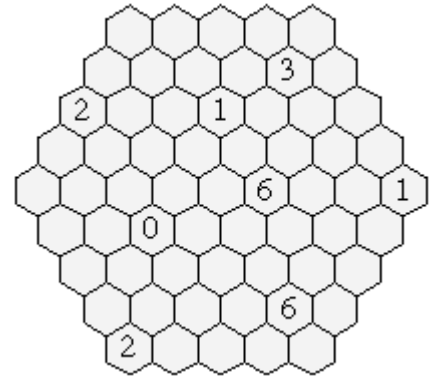

Puzzle 10

of every one of its thousands of members rows and columns on the wall. Because decided to re-arrange them. "If we have "we will have 42 less pictures in each have four more rows, we will have 76 less does the social club have?

arrangements could the club choose from? square?

Intermediate Puzzle 11

The diagram on the left shows a hexagonal grid with a closed path which travels from hexagon to hexagon, never making a **sharp** turn. The hexagons through which the grid does **not** pass contain numbers showing the number of **adjacent** hexagons through which the path **does** pass. On the right is another hexagonal grid with a hidden path of the same type. Some of the hexagons through which the grid does not pass are marked with numbers in the same way as for the grid on the left. Can you reconstruct the path and fill in the remainder of the numbers?
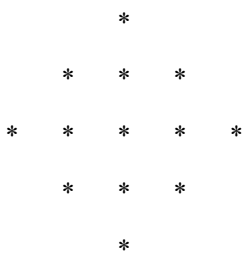

Intermediate Puzzle 12
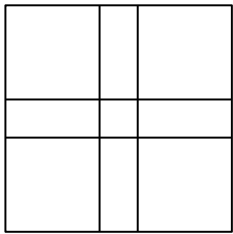
Alice and Bea are making a flag. It is to be a green rectangle 20cm by 21cm, with a white cross on it. The horizontal and vertical bars of the cross should be the same width. To symbolise equality, they want to make the area of green and the area of white exactly equal. How wide should the bars of the cross be?

Extension: If the rectangle is a cm by b cm, how wide should the bars of the cross be?

Intermediate Puzzle 13

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        *                                                              *   *   *
    *   *   *                                                          *   *   *
*   *   *   *   *                                                      *   *   *
    *   *   *
        *
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Without taking your pencil from the paper, can you draw five straight lines which pass through all the thirteen stars in the diagram on the left?

[This is a variation on the old puzzle about drawing four straight lines through the nine stars in the diagram on the right - if you haven't tried this one, you'd better try that before you move on to the main puzzle.]

Intermediate Puzzle 14

Anna, Beth and Caroline had an unusual combination of ages. The sum of any two of their ages was equal to the third age with the digits reversed. All were under 100 years old, and no two had the same age. What were their ages?
(Don't just use trial and error - you should be able to get this using logic and/or algebra.)

Intermediate Puzzle 15

Tony handed back the photo "So that's your two boys and Alice, eh?" he commented. "She must be a teenager now."

"That's right. Their ages make a neat puzzle for you. The reciprocal of the square of her age is equal to the difference between the reciprocals of the squares of the ages of her brothers."
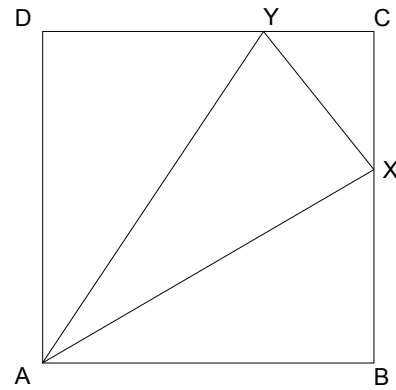
How old is Alice?

[The reciprocal of x is $\frac{1}{x}$. All three ages are whole numbers.]

Intermediate Puzzle 16

ABCD is a square. X and Y are points on the sides of the square such that XY = 3cm, AX = 4cm and AY = 5cm
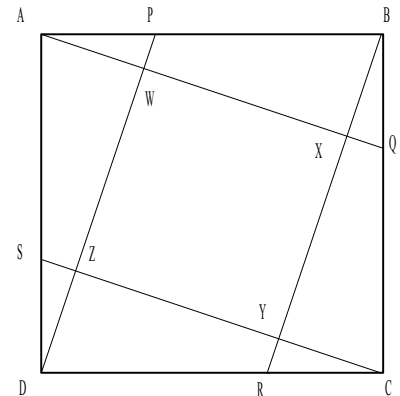
Find the length of a side of the square.

[Hint: leave your answer in the form $\sqrt{\dfrac{p}{q}}$]

Intermediate Puzzle 17

In this diagram, AP= $\dfrac{1}{3}$ AB, BQ= $\dfrac{1}{3}$ BC etc. What fraction of the area of ABCD is the area of WXYZ?

What if AP= $\dfrac{1}{n}$ AB, etc?

Intermediate Puzzle 18

Consider this triangle of numbers, in which the nth diagonal line consists of the multiples of n. For example, the third diagonal line is 3, 6, 9, 12, etc. As you can see, the number 12 first appears on the 6th horizontal row.

In which horizontal row will the number 77 first appear?
In which horizontal row will the number 105 first appear?
How many times will 105 appear in the triangle?

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                    1
                 2     2
              3     4     3
           4     6     6     4
        5     8     9     8     5
     6    10    12    12    10    6
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Intermediate Puzzle 19

Thirty years ago the ages of Xavier, Yolande and Zoë were in the ratio 1:2:5. Today, the ratio of Xavier's age to Yolande's age is 6:7. What is Zoë's present age?

Intermediate Puzzle 20

The sides of a right angled triangle form an arithmetic series (ie the difference in length between the smallest side and the middle side is equal to the difference between the middle side and the longest side).

Prove that the ratio of the three lengths must be 3:4:5.

[Hint: Call the length of the middle side *x* and form a quadratic equation.]

Intermediate Puzzle 21

If I start with the number 123, and place a 5 at each end of the number, I get 51235. So the original number has been increased by 51112.

Find the positive integer which is increased by 518059 when a 5 is placed at each end of the number.

Intermediate Puzzle 22

The head of a school plans to employ more teachers in order to reduce class sizes. If she employs 10 new teachers, then the number of pupils per teacher in the school will be reduced by 5. But if she employs 20 new teachers, then the number of pupils per teacher will be reduced by 8.

How many pupils are there in the school? (You need to explain how you worked this out.)

Intermediate Puzzle 23

Show that there is only one set of five consecutive positive integers such that the sum of the squares of the first three integers is equal to the sum of the squares of the last two integers.

Intermediate Puzzle 24

The diagram shows two concentric circles. The length of a line which is a chord of the larger circle and a tangent to the smaller circle is 10cm.

What is the area of the shaded part of the diagram?

10 cm
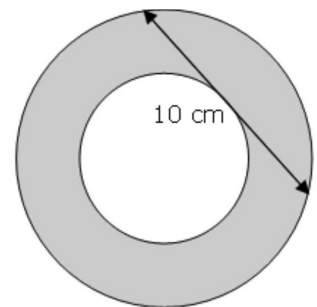
Intermediate Puzzle 25

Alice was on her way to her grandmother's house when she came to a wooden bridge across a stream. As she approached it, out popped a troll. "Halt" he said, "To cross this bridge you must pay my fee."

"How much?" asked Alice. "That depends," said the troll. "The first two people to cross each morning just have to pay me all the money in their pocket. After that, each person has to pay as much as the two previous people added together."   "But that could be lots and lots," cried Alice, "I might not have that much. How many people have already crossed today?"

"You will be the sixth – if you cross," said the troll smiling to himself. "And the first person only had £2." "It's no use – I still can't work it out," said Alice. "All right," said the troll, "I'll tell you. You'll have to pay £17." Alice looked in her purse, and found she had just enough. "But I'll get here at daybreak next time, with two pence in my pocket!" she thought.

After Alice pays him, how much money will the troll have taken in all today?

Intermediate Puzzle 26

After visiting my Aunt Hilda, I set off home in my old banger. I had filled up with petrol, so I knew I had 60 litres in my tank. But I also knew that the tank leaked 2 litres of fuel per hour, in addition to the fuel used by the engine. And because the car was stuck in second gear, I could only go at a speed of 30mph, and only travelled 5 miles per litre of petrol. So all in all I was worried about whether I would get home before the fuel ran out. As it happened it ran out just as I arrived outside my house. How many miles was it from Aunt Hilda's house to mine?

Intermediate Puzzle 27

The elves in Santa's workshop began making rocking horses straight after last Christmas and kept making them for 52 weeks. They gradually became able to make them more quickly, so that in the second week, they made one more rocking horse than in the first week, in the third week they made one more than in the second week, and so on, until in the 52nd week they made 1 more than in the 51st week. When they counted up how many rocking horses they had made altogether, they found they had 2210. How many did they make in the first week?

[You will need to use algebra to solve this. Dr Secker's Brain Gym might help!)

Intermediate Puzzle 28
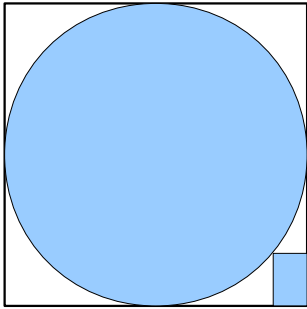
This diagram shows a square with sides 10cm, with a circle of radius 5 inside it. A rectangle in the corner of the square just touches the circle at one vertex. The height of the rectangle is twice its width.

What fraction of the area of the square is the area of the rectangle?

Explain how you work this out.
[Hint: You will need to solve a quadratic equation.]

Intermediate Puzzle 29

In this shape, P, Q R and S are four points on a line such that PQ = RS.
Semicircles are drawn above the line with diameters PQ, RS and PS.
Another semicircle with diameter QR is drawn below the line.
The line MN is the line of symmetry of the figure.

Prove that the shaded area is equal to the area of a circle with diameter MN.
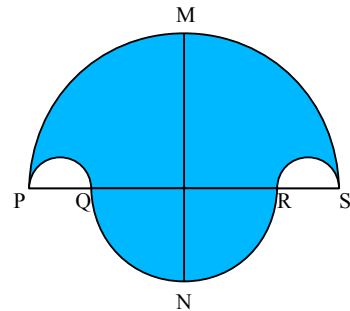
Intermediate Puzzle 30

At Cambridge Low School there are 300 pupils who all play a summer sport and a winter sport. In summer 60% of pupils play tennis and the remainder play badminton. In winter they either play hockey or swim (but not both). 56% of the hockey players play tennis in summer. 30% of the tennis players swim. How many of the students play badminton in summer and swim in the winter?

Intermediate Puzzle 31

In my town, 10% of the dogs think they are cats, and 10% of the cats think they are dogs. All the other cats and dogs are perfectly normal. When all the cats and dogs in my town were rounded up and subjected to a rigorous test, 20% of the animals thought they were cats. What percentage of the animals really were cats?

Intermediate Puzzle 32

When Ernie was as old as Bert is now, Bert's age was half of Ernie's present age. When Bert gets to be as old as Ernie is now, the sum of their ages will be 108. How old are Ernie and Bert at the moment?
[The solution can be obtained using algebra. To get a prize you must explain how you find your solution.]

Intermediate Puzzle 33

The social club had a passport photo of every one of its thousands of members displayed in a rectangular array of rows and columns on the wall. Because the rows were getting too long, they decided to re-arrange them. "If we have two more rows," said one member, "we will have 42 less pictures in each row." "Yes,", said another, "but if we have four more rows, we will have 76 less in each row." How many members does the social club have?

[Extensions: How many rectangular arrangements could the club choose from?   Which arrangement is closest to a square?]

Intermediate Puzzle 34

There is a cheese shortage, and everyone is stocking up. By the time Ann and Belinda get to the shop, there are only 10 pre-packed lumps of cheese left, with weights of 15, 16, 18, 19, 23, 24, 25, 27, 28, 31 and 37 ounces. Between them Ann and Belinda buy up all the remaining cheese, except for one lump which looks a bit mouldy. When they get home, they find that the total weight of Belinda's cheese is exactly twice that of Ann's cheese. What was the weight of the mouldy lump?
[Hint: There is no need for any trial and error here - you do not need to work out who bought which pieces of cheese. But you do need to explain the logic you use to work it out.]

Intermediate Puzzle 35

Our family always likes to have some nuts to nibble at Christmas. Unfortunately everybody thought of this on the same day. So Dad came home with a pound of walnuts and two pounds of Brazil nuts, which cost him £5. Mum came in with four pounds of Cashews and a pound of walnuts, which cost £6, and Aunt Agatha contributed three pounds of almonds, a pound of walnuts and a pound of cashews, which cost £3. Lastly I arrived with one pound of each of the four kinds of nut. How much did they cost me? Explain how you worked it out.

Intermediate Puzzle 36

How many six digit odd integers can be made by rearranging the numbers 0,1,2,3,4 and 5?
[Hint: don't just try to list them all – there are quite a lot. You need to find and explain a method for working this out.
Also, an integer can't start with zero – 012345 is not a 6 digit integer, because it should be written 12345.]

Intermediate Puzzle 37

Place a number in each cell of this three by three grid, so that in each row and each column, the sum of the three numbers is 7, and in each 2-by-2 square within the grid, the sum of the four numbers is 12.

[You must explain how you work this out – do not just use trial and error.]