# Algebra Puzzle 1

“How old are you, Aunt Alice?” asked Bobby. “Five times as old as you,” she replied. “That's OLD!” Bobby exclaimed. “Ah,” said Aunt Alice, “but in 24 years time I will only be twice as old as you – I won't be so old then, will I?” “You will,” said Bobby, “but I'll be old too!”. How old are Bobby and Aunt Alice at the moment?
The solution can be obtained using algebra. Try to explain how you find your solution.

# Algebra Puzzle 2

When Ernie was as old as Bert is now, Bert's age was half of Ernie's present age. When Bert gets to be as old as Ernie is now, the sum of their ages will be 108. How old are Ernie and Bert at the moment?
The solution can be obtained using algebra. Try to explain how you find your solution.

# Algebra Puzzle 3

After visiting my Aunt Hilda, I set off home in my old banger. I had filled up with petrol, so I knew I had 10 gallons in my tank. But I also knew that the tank leaked half a gallon of fuel per hour, in addition to the fuel used by the engine. And because the car was stuck in second gear, I could only go at a speed of 30mph, and only travelled 20 miles per gallon of petrol. So all in all I was worried about whether I would get home before the fuel ran out. As it happened I ran out just as I arrived outside my house. How many miles was it from Aunt Hilda’s house to mine?

# Algebra Puzzle 4

The elves at the Lapland Weather Centre have been keeping records for quite a few days this winter. They have found that whenever it snowed in the afternoon, it had been fine in the morning, but whenever it snowed in the morning, it swas fine in the afternoon. So far it has snowed on 100 days, and been fine on 19 afternoons and 95 mornings. Forhow many days have they been keeping a record? Try to explain how you work it out.

# Algebra Puzzle 5

Miss Speed has left it rather late to get to work on time. Driving along, she works out that if she averages 45 miles per hour, she will make it with a minute to spare. If, on the other hand, she only averages 40 mph, she will be one minute late. How far does she have to travel?

# Algebra Puzzle 6

A mixed packet of nuts containing 1kg of walnuts and 2kg of brazil nuts costs £1.98. A packet containing 4 kg of hazelnuts and 1 kg of walnuts costs £2.48. And for £2.99 you can buy a mixed packet of 3kg of almonds, 1kg of walnuts and 1kg of hazelnuts. How much should you pay for a bag containing 1kg of each of the four types of nut?

# Algebra Puzzle 7

I have drawn a rectangle. If I increase the length by 1cm and decrease the width by 1cm, the area of the rectangle will increase by 5 $cm^2$. What will happen to the area if I decrease the length by 1cm and increase the width by 1cm? Prove that your answer is always true.

# Algebra Puzzle 8

The sequence 3, 7, 10, 17, 27, 44, 71, 115, 186, ... is formed using the rule that, apart from the first two terms, each term is the sum of the two preceding terms. I notice that if I add together the $6^{th}$ and the $9^{th}$ terms, I get twice the $8^{th}$ term ($44 + 186 = 2 \times 115$). Try this with a different sequence formed using the same rule. Prove that it will work for any sequence formed using this rule. Can you generalise this further?

# Algebra Puzzle 9

A website invites you to give the stories on it a rating from 1 to 5, and displays the average (mean) rating for each story. One story had an average rating of 2.6, but I read it and really liked it, so I gave it a rating of 5. The average rating then changed to 2.75. How many people had rated the story before I did? Show your method.

# Algebra Puzzle 10

A man walks up a moving escalator, taking one step per second. After taking thirty steps, he is at the top.
Next day, he goes up at two steps per second, reaching the top after taking 36 steps.
The third day, he has had a long afternoon and merely stands on the escalator, waiting for it to reach the top.
How many seconds does it take him? Show how you work out the answer.

# Algebra Puzzle 11

A sequence is formed by using the rule that, apart from the first two terms, each term is the sum of all the preceding terms. The 8th term is 212. What is the fourth term? Show how you find your answer.

# Algebra Puzzle 12

```
    A
  MERRY
+  XMAS
 TURKEY
```

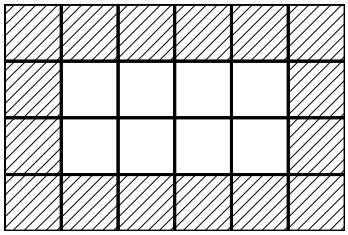
In the addition sum on the left, each letter stands for a different digit (so if A is 5, M cannot also be 5). A letter stands for the same number everywhere it occurs (so if the M in "MERRY" is 5, then the M in "XMAS" must also be 5. Can you work out what each letter stands for?

# Algebra Puzzle 13

In a puzzle challenge, three problems A, B and C were posed. Among the contestants there were 58 who solved at least one problem correctly. Of all the contestants who did not solve problem A, the number who solved B was twice the number who solved C. The number of participants who solved only problem A was one more than the number who solved problem A and at least one other problem. Of all students who solved just one problem, half solved problem A.

How many students solved only problem B?

# Algebra Puzzle 14



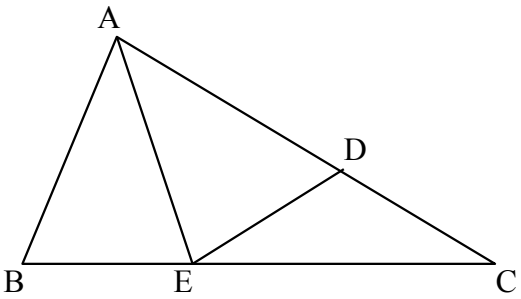
Al was planning to lay a rectangular patio in his garden, using square paving slabs. He wanted to have one row of patterned slabs all round the edge, with plain slabs in the middle, as in the diagram (though this diagram is not the correct size). When he worked out how many of each sort he needed to order, he found that the number of patterned slabs was $\frac{1}{3}$ of the total number of slabs. How many slabs long and how many slabs wide was his patio? [Note: there are several possible sizes for the patio – try to find a systematic way to find them all; remember this is the algebra puzzle!]

Extension: If the number of patterned slabs is $\frac{1}{n}$ of the total, can you give a possible formula in terms of n for the length and width of the patio (there are several solutions to this part too).

# Algebra Puzzle 15

You are sent to the market by your father with £800, and told to buy 100 animals. When you arrive at the market, you find out that pigs cost £8 each, so it would be easy to follow your father's instructions by buying 100 pigs.
However, you find that there are only 99 pigs for sale. The only other animals there are chickens for £1 each, and cows for £80 each.
If there are enough chickens and cows for you to buy as many as you like, how can you still end up buying 100 animals using exactly £800? Explain how you work this out.

# Algebra Puzzle 16




This diagram (which is not drawn accurately) shows a triangle ABC with extra lines AE and DE inside it. If lines AB, AE, DE and DC are all the same length, what is the relationship between angles B and C in the triangle? Make sure you justify your answer.

# Algebra Puzzle 17

Alice was on her way to her grandmother's house when she came to a wooden bridge across a stream. As she approached it, out popped a troll. "Halt" he said, "To cross this bridge you must pay my fee."

"How much?" asked Alice. "That depends," said the troll. "The first two people to cross each morning just have to pay me all the money in their pocket. After that, each person has to pay as much as the two previous people added together." "But that could be lots and lots," cried Alice, "I might not have that much. How many people have already crossed today?"

"You will be the sixth – **if** you cross," said the troll smiling to himself. "And the first person only had £2." "It's no use – I still can't work it out," said Alice. "All right," said the troll, "I'll tell you. You'll have to pay £17." Alice looked in her purse, and found she had just enough. "But I'll get here at daybreak next time, with two pence in my pocket!" she thought.

After Alice pays him, how much money will the troll have taken in all today? Explain how you work this out.

# Algebra Puzzle 18

Sarah was out walking in the countryside when she came to a wooden bridge across a stream. As she approached it, out popped a troll. "Halt" he said, "I am the Double Crossing Troll. If you cross this bridge, I will double the money you have in your pocket. But then you must pay me my fee."

"How much?" asked Sarah, and the troll told her. Sarah checked the money in her pocket. "OK" she said, and marched bravely across the bridge. When she got to the other side, she felt in her pocket, and sure enough her money had doubled. She threw the troll his fee. She was about to go on her way, when a thought occurred to her. She walked back across the bridge, and checked her pocket. Yes, her money had doubled again. So she threw the troll his fee, and crossed for a third time. Again her money doubled, and after paying the troll again, she counted it up. "Right," she thought, "I've now got exactly twice as much as I started with. So if I just keep crossing the bridge all afternoon…."

But then she remembered all the fairy tales she had read about what happened to people who were too greedy. "All right," she thought "I'll just cross 3 more times, and then go home." And that is what she did – each time, her money doubled, and then she paid the troll. When she got home, she counted her money, and found she had … how many times what she had set off with?

# Algebra Puzzle 19

Alan throws three darts at a dartboard, and hits three of the numbers from 1 to 20. The total of these three numbers is 31. However, one of the darts lands in the "double" section, and one in the "treble" section, while the third landed in the "single" section, so his overall score for the three darts is 64. Prove that the number of the dart which hit the "treble" was 2 more than the number which hit the "single" section.

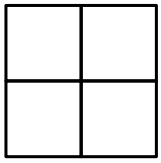
# Algebra Puzzle 20

I think of five numbers a, b c d and e. When I add them in every possible pair combination (a + b, a + c, etc) I get the values: 0,1,2,4,7,8,9,10,11,12. What are my original 5 numbers? (Try to use algebra rather than trial and error!)

# Algebra Puzzle 21

Alice has been collecting her small change (1p, 2p and 5p coins) to give to charity. When she counted it up, she found that she had exactly 100 coins. There were twice as many 1p coins as 5p coins, and the total amount was £2.24. How many of each type of coin did she have?

# Algebra Puzzle 22



Place numbers into this square so that:

- The sum of all four numbers is 340
- The sum of the two numbers in the first column is twice the sum of the numbers in the first row
- The sum of the numbers in the second row is three times the sum of the numbers in the second column
- The sum of the two numbers on each diagonal is equal

# Algebra Puzzle 23

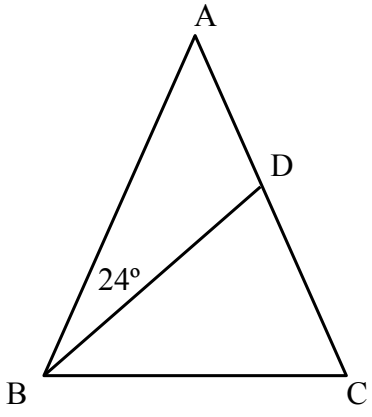
Mr and Mrs Series have had 5 children at regular intervals, so that their ages are all equally spaced. The sum of the ages of the two oldest children is equal to the sum of the ages of the three youngest children. What is the ratio of the ages of the eldest and the youngest child?

# Algebra Puzzle 24

I am thinking of three numbers, all different. The sum of all three numbers is 8 times the smallest number. The sum of the largest number and the smallest number is 3 times the middle number. How many times larger than the smallest number is the largest number? Show how you worked this out.

# Algebra Puzzle 25

In this diagram, AB = AC and BC = BD. $A\hat{B}D$ = 24º. Find $B\hat{A}C$
[Hint: Why is this an "algebra" puzzle?]




# Algebra Puzzle 26

Three of Santa's elves (Arwel, Bronwen and Caedmon) were counting how many mince pies they had collected while helping Santa deliver his presents. They decided to even them out among them. First, Bronwen gave Arwel as many pies as he (Arwel) already had. Then, Caedmon gave Bronwen as many pies as she (Bronwen) had left after giving some to Arwel. Then, Arwel gave Caedmon as many pies as Caedmon had left after giving some to Bronwen. After all this, they discovered that they each had the same number of pies. If they started with 240 pies between them, how many did each one start with?

# Algebra Puzzle 27

Here is a number trick: Think of a three digit number, with all digits different. Now make all the possible two digit numbers which can be formed from the digits of your number (there should be six of them). Add together these six numbers. Now divide by the sum of the digits of your original three digit number. The answer you get is...?

The puzzle is: Prove that you will always get the same answer, no matter what 3 digit number you start with.

[Hint:
In algebra, the two digit number formed from the digits x and y is written "10x + y" - not "xy", which of course means x × y.]

# Algebra Puzzle 28

I am thinking of a three digit number with three different digits. I form all the possible three digit numbers which can be formed from the digits of my number, not counting my original number (there are five of them). I add up my five rearrangements and find the total is 3961. What was my original number?
[Hint: use the same type of method as for puzzle 27.]

# Algebra Puzzle 29

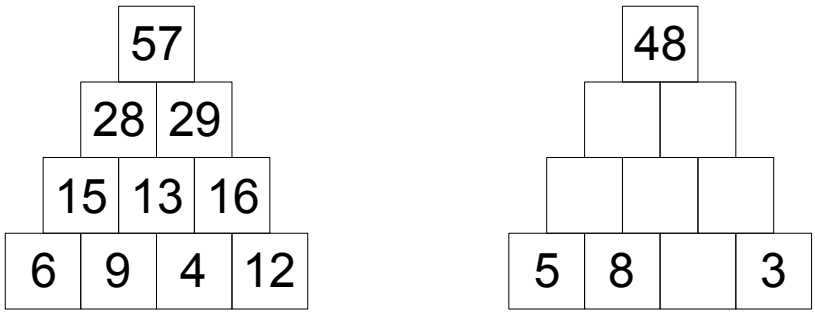
The multiplication sum 23 × 64 = 1472 is unusual because if we reverse the order of the digits in the two two-digit numbers being multiplied, we get 32 × 46, but the answer is still 1472.
Find all the two-digit multiplications which work in this way.
If you can't work out a way to find them all, find as many as you can.
[Hint: The same technique as was used in the previous two puzzles will work again here.]

# Algebra Puzzle 30

Little Jane is playing with the balancing scales from her kitchen. She is putting the plastic animals from her toy zoo onto the scales to see what will balance. She finds that:
one elephant and one giraffe will balance three zebras
one giraffe and one zebra will balance four antelopes
one antelope and two zebras will balance one elephant

How many antelopes would it take to balance an elephant?

# Algebra Puzzle 31

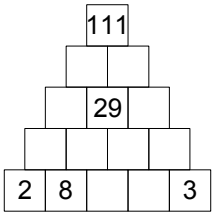



On the left is an example of a "number pyramid", in which the number in each box is the sum of the numbers in the two boxes on which it rests. (That is, except for the boxes on the bottom!)

Can you work out the missing number on the bottom row of the pyramid on the right? Explain how you worked it out.

[Hint: It is not a whole number. You will need to use algebra to find it.]

# Algebra Puzzle 32




Another pyramid puzzle this month.. (Look at last month's algebra puzzle to see how the “number pyramid” works.)

Can you work out the two missing numbers in the bottom row of this pyramid?

Show how you worked them out.

# Algebra Puzzle 33

| | | |
|---|---|---|
| 8 | | |
| | | 6 |
| 11 | | |

This is a magic square – so the total of the three numbers in each row, column and main diagonal is the same.

Can you fill in the rest of the square?

[Hint: either let the total be t, and work out some of the values in the square in terms of t. Or let the numbers in the middle row be x, y and 6, and work it out from there.]

# Algebra Puzzle 34

Little Mark is playing with the animals on his toy farm, placing them on his mother's scales and trying to make them balance. He finds that 2 cows will balance with 3 sheep and 4 hens. Also 2 sheep will balance with 1 cow and 1 hen. How many hens would it take to balance one cow and one sheep?

# Algebra Puzzle 35

Each day a man has to travel up an escalator on his way home from work.
On Monday, he walked up, taking 1 step per second. After taking 30 steps, he reached the top.
On Tuesday, he ran up, taking 2 steps per second. After taking 36 steps, he reached the top.
On Wednesday, he was tired, so he just stood on the escalator and let it take him up. How many seconds did it take him to reach the top?

# Algebra Puzzle 36

Can you work out how to assign values to each of the letters N, O , U , G, H, T, E, W, R, F, I, V, S, X, and L, so that the sum of the letters in each of the words NOUGHT, ONE, TWO, THREE, up to FIFTEEN is equal to the value which the word represents?
(ie N + O + U + G + H + T = 0, O + N + E = 1 etc) Some letters will have the same value.

# Algebra Puzzle 37

Each month, the sunflower which I am growing in my garden increases in height by an amount equal to its height at the start of the previous month. (For example, if its height was 10cm at the beginning of April, and 15cm at the beginning of May, then by the beginning of June it would have grown 10cm and be 25cm tall.)
At the beginning of June 2006 it was 25cm high, and at the beginning of September 2006 it was 106cm high. What was its height at the beginning of January 2006?

# Algebra Puzzle 38

There are 525 students in a school. The *number* of these who are sixth formers is equal to the *percentage* of pupils who are not sixth formers. How many sixth formers are there?

[Hint: You will need to use algebra to solve this.]

# Algebra Puzzle 39

Given that a, b, $x$, d and e are 5 consecutive integers, and $a^2 + b^2 + x^2 = d^2 + e^2$, find the values of a, b, $x$, d and e.

[Hint: **Don't** just use trial and error. Use algebra; there is a hint in the letters I have used.]

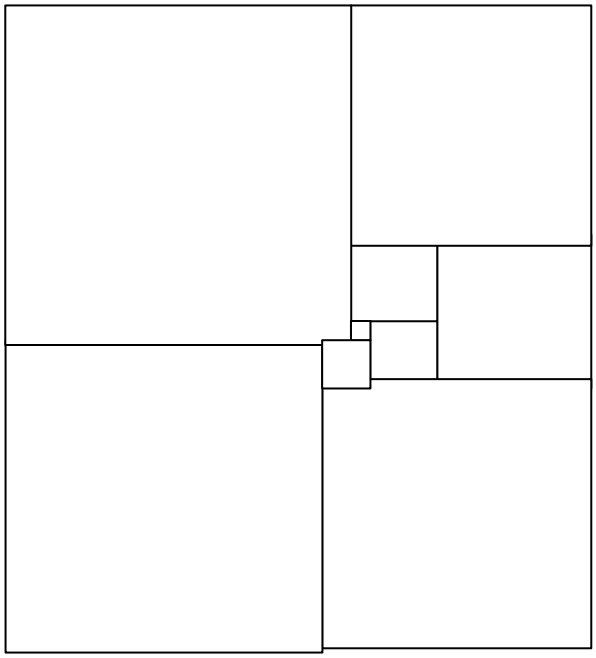
# Algebra Puzzle 40

I met two of Santa's elves the other day – Rupert and his sister Ruperta. "We elves have very large families," Rupert told me. " I have three times as many brothers as sisters." "And how do you feel about having three times as many brothers?" I asked Ruperta. "Don't be silly – I have **five** times as many brothers as sisters," she replied. I looked puzzled for a minute. "Surely now you can work out now how many boys and how many girls there are in our family," she asked. Can you?
[Remember to show how you do this using algebra.]

# Algebra Puzzle 41

A girl cycles along a straight road to see her grandmother. Because the wind is against her, the journey takes 4 hours. On the way back, with the wind assisting her, she does the journey in 3 hours. How long would the journey take if there were no wind? Explain your method of working this out.

# Algebra Puzzle 42



This diagram shows a **rectangle** divided up into a number of **squares** of different sizes. The smallest square has sides x cm, and the next smallest has sides y cm long. If the sides of the rectangle are the smallest possible whole numbers, what must the values of x and y be and what is the size of the rectangle?

[This puzzle illustrates part of the method used by a group of mathematicians who discovered in the 1930's a method of "squaring the square" – dividing up a **square** into a number of smaller squares, all of different sizes, with lengths that are whole numbers. The smallest known such square is made up of 24 smaller squares.]

# Algebra Puzzle 43

```
  T   E   A   S   E   R
    S   T   U   D   Y
```

This TEASER is odd, so it will repay careful STUDY. Different letters stand for different non-zero digits. Each letter on the bottom line is the sum of the two letters above it (for example U = A + S). What RESULT do you come up with?

# Algebra Puzzle 44

400 sweets are shared out between Ann, Briony, Chloe and Diana, so that the Ann's share multiplied by 4, Briony's share divided by 4, Chloe's share plus 4, and Diana's share minus 4, all equal the same number. What is each person's share? Explain how you worked it out.

# Algebra Puzzle 45

Our family always likes to have some nuts to nibble at Christmas. Unfortunately everybody thought of this on the same day. So Dad came home with a pound of walnuts and two pounds of Brazil nuts, which cost him £5. Mum came in with four pounds of Cashews and a pound of walnuts, which cost £6, and Aunt Agatha contributed three pounds of almonds, a pound of walnuts and a pound of cashews, which cost £3. Lastly I arrived with one pound of each of the four kinds of nut. How much did they cost me? Explain how you worked it out.

# Algebra Puzzle 46

To encourage children to participate in sport, the governors of a school decided to give each girl at the school £12 worth of vouchers for entry to the local sports centre, and to give each boy £8 worth of vouchers. However, of the 612 students, only half of the girls and ¾ of the boys took up the offer. What was the total value of the vouchers given out?

# Algebra Puzzle 47

Alice has been collecting her small change (1p, 2p and 5p coins) to give to charity. When she counted it up, she found that she had exactly 100 coins. There were twice as many 1p coins as 5p coins, and the total amount was £2.24. How many of each type of coin did she have?

# Algebra Puzzle 48

Isobel has a large number of boxes in three sizes: large, medium, and small.
She puts 11 large boxes on a table. She leaves some of these boxes empty, and into each of the other boxes she puts 8 medium boxes. She leaves some of these medium boxes empty, and into each of the other medium boxes she puts 8 (empty) small boxes.
Now 102 of ***all*** the boxes on the table are empty.
How many boxes has Isobel used in total?