

Recurring and non-recurring decimals

Depending on the value of n , the decimal for $\frac{1}{n}$ can take one of three forms:

Type A: It may terminate after a certain number of digits

for example $\frac{1}{5}=0.2$ $\frac{1}{16}=0.0625$ $\frac{1}{25}=0.04$

Type B: It may recur after a certain number of digits

for example $\frac{1}{3}=0.\overline{3}$ $\frac{1}{11}=0.\overline{09}$ $\frac{1}{7}=0.\overline{142857}$

Type C: It may be "hybrid" ie start with some non-recurring digits, then continue with a certain number of recurring digits

for example $\frac{1}{12}=0.08\overline{3}$ $\frac{1}{55}=0.0\overline{18}$ $\frac{1}{296}=0.003\overline{378}$

The question is, for a given value of n , how can we tell whether $\frac{1}{n}$ will be of type A, type B or type C?

Questions

- 1) What can you say about n if $\frac{1}{n}$ is Type A (ie it terminates)? Can you explain your answer?
- 2) By factorising n , can you tell how many digits $\frac{1}{n}$ will have before it terminates?
- 3) Some specific questions for which you can check your answers:
 - (a) There are 7 values of n for which $\frac{1}{n}$ terminates after 3 digits. Can you find them all?
 - (b) For how many values of n will $\frac{1}{n}$ terminate after 30 digits?
- 4) What type of number must n be for $\frac{1}{n}$ to be of Type B?
- 5) Some more questions for which you can check your answers:
 - (a) For how many values of n does $\frac{1}{n}$ consist of just 3 recurring digits? What are these values?
 - (b) There are 6 values of n for which $\frac{1}{n}$ consists of just 4 recurring digits. What are these values?
- 6) What is the maximum number of recurring digits which $\frac{1}{n}$ can have? Explain why this is.
- 7) List the values of n up to 20 which achieve this maximum value.
- 8)
 - (a) What type of number must n be for $\frac{1}{n}$ to achieve its maximum number of digits?
 - (b) Do all values of n of this type achieve this maximum?
 - (c) If not, what can you say about the number of recurring digits?
- 9) What type of number must n be for $\frac{1}{n}$ to be of Type C?
- 10) There are 9 values of n for which $\frac{1}{n}$ has 1 non-recurring digit followed by 2 recurring digits.
- 11) If $\frac{1}{m}$ is Type B with x recurring digits, and $\frac{1}{n}$ is Type B with y recurring digits, what can you say about the number of recurring digits in $\frac{1}{mn}$? Explain why.
- 12)
 - (a) Go back to question 8 and prove the answers you gave there.
 - (b) For which primes p does $\frac{1}{p}$ achieve the maximum number of recurring digits? Recurring and non-recurring decimals?