

## Algebra Revision 1 – Answers

1) (a)  $5p^2 - 15pq$       (b)  $9a + 4b$       (c)  $3x^2 - 2xy - 8y^2$

2) (a)  $3(5p - 3p)$       (b)  $x(x + 5)$       (c)  $4a(2a - 3b)$

3) (a)  $(x + 4)(x + 7)$       (b)  $(x - 10)(x - 3)$       (c)  $x - 15)(x + 2)$

4) (a)  $(x + 7)(x - 7)$       (b)  $(x + 10)(x - 10)$

5) (a)  $x = -1.5$       (b)  $x = 18$

6) (a)  $x = 3, y = 2.5$       (b)  $x = 6, y = -5$       (c)  $x = 4, y = -1$

7) (a)  $\frac{x+7}{x-2}$       (b)  $\frac{x+5}{x-3}$

8) (a)  $x = 5$  or  $x = 6$       (b)  $x = 10$  or  $x = -2$

9) (a)  $x = 3$  or  $x = 0.5$       (b)  $x = 3.14$  or  $x = -0.64$

10) Draw diagram and divide up the path into sections to find that:  
Area of path =  $4x^2 + 20x$

So  $4x^2 + 20x = 96$

Divide by 4:  $x^2 + 5x = 24$

Subtract 24 from each side:  $x^2 + 5x - 24 = 0$

$$(x + 8)(x - 3) = 0$$

Either  $x + 8 = 0$  or  $x - 3 = 0$

$x = -8$  (not suitable for width of path) or  $x = 3$

## Algebra Revision 2 – Answers

1) (a) (i) 
$$\frac{x^2 - 4x}{x^2 - 16} = \frac{x(x-4)}{(x+4)(x-4)}$$

$$= \frac{x}{x+4}$$

(ii) 
$$\frac{3}{x-2} - \frac{2}{x+5} = \frac{3(x+5) - 2(x-2)}{(x-2)(x+5)}$$

$$= \frac{3x+15-2x+4}{(x-2)(x+5)}$$

$$= \frac{x+19}{(x-2)(x+5)}$$

(iii) 
$$\frac{2}{x-2} - \frac{1}{x^2-4} = \frac{2(x^2-4) - 1(x-2)}{(x-2)(x^2-4)}$$

$$= \frac{2x^2-8-x+2}{(x-2)(x^2-4)}$$

$$= \frac{2x^2-x-6}{(x-2)(x^2-4)}$$

$$= \frac{(2x+3)(x-2)}{(x-2)(x^2-4)}$$

$$= \frac{2x+3}{x^2-4}$$

(b) (i)  $(x^{1.5})^2 = x^3$       (ii)  $\frac{x^{\frac{2}{3}} \times x^{\frac{1}{3}}}{x^{-2}} = x^{1-(-2)} = x^3$       (iii)

$\sqrt{x^{10}} = (x^{10})^{\frac{1}{2}} = x^5$

2) Rearrange each of these formulae to make r the subject:

(i)  $S = 2ar + c$   
 $S - c = 2ar$   
 $\frac{S-c}{2a} = r$

(ii)  $d = \frac{2ab}{r}$   
 $dr = 2ab$   
 $r = \frac{2ab}{d}$

(iii)  $A = 2r^2$   
 $\frac{A}{2} = r^2$   
 $\sqrt{\frac{A}{2}} = r$

(iv)  $M = \frac{1+ar}{1-r}$   
 $M(1-r) = 1+ar$   
 $M - Mr = 1+ar$   
 $M - 1 = ar + Mr$   
 $M - 1 = r(a + M)$   
 $\frac{M-1}{a+M} = r$

3)  $y = kx^2$   
 $20 = k \times 4^2$   
 $k = \frac{20}{16} = \frac{5}{4}$   
 So  $y = \frac{5}{4}x^2$   
 When  $x = 6$ ,  $y = \frac{5}{4} \times 6^2 = 45$

4)  $y = \frac{k}{\sqrt{x}}$   
 $40 = \frac{k}{\sqrt{25}}$   
 $40 \times 5 = k$   
 $k = 200$   
 So  $y = \frac{200}{\sqrt{x}}$

When  $x = 256$ ,  $y = \frac{200}{\sqrt{256}} = 12.5$

5) (a)

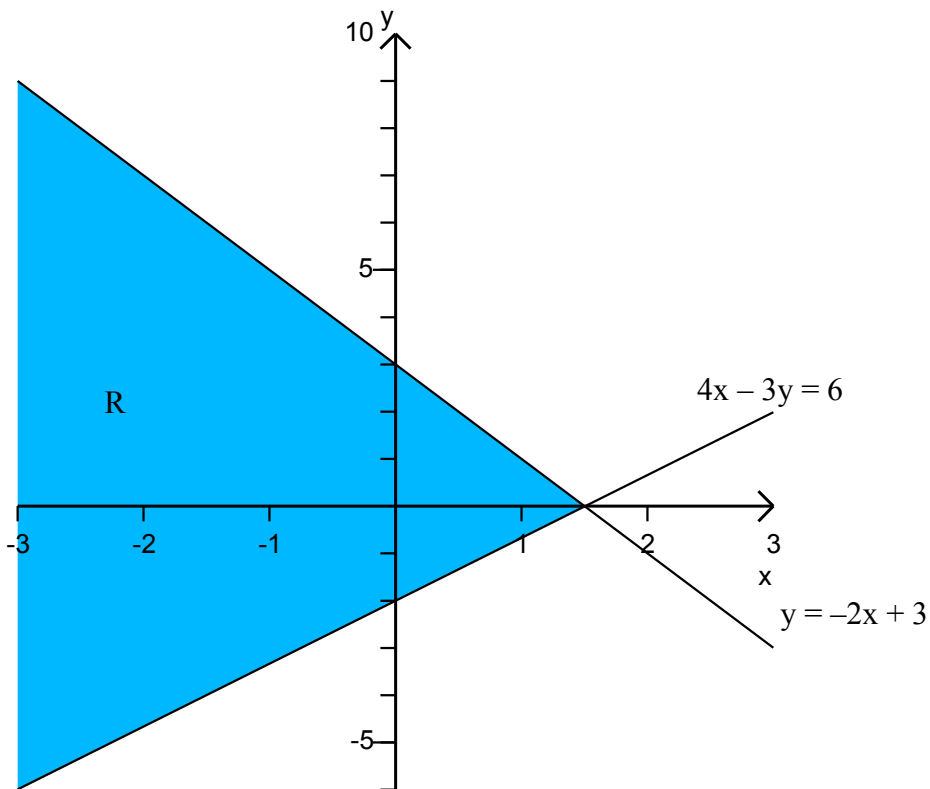
x	-3	-2	-1	0	1	2	3
y	-6	$-4\frac{2}{3}$	$-3\frac{1}{3}$	-2	$-2\frac{2}{3}$	$2\frac{2}{3}$	2

(b) See below

(c)  $\frac{4}{3}$

(d)  $y = \frac{4}{3}x - 2$

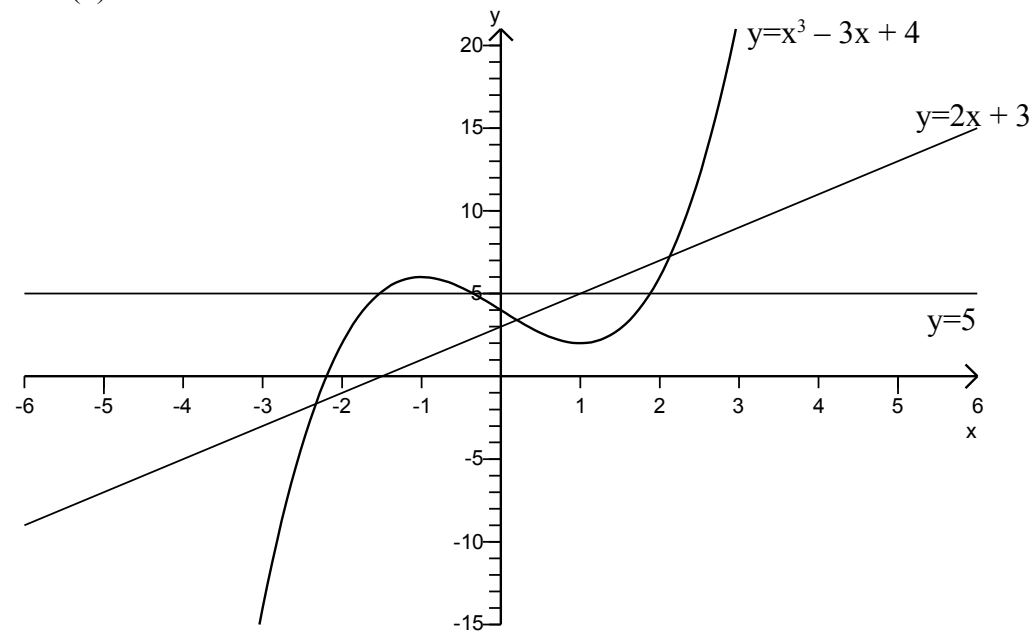
(e) & (f)



8) (a)

x	-3	-2	-1	0	1	2	3
y	-14	2	6	4	2	6	22

(b)



(c) (i) Draw the line  $y = 5$ .

$$x = -1.5 \text{ or } x = -0.3 \text{ or } x = 1.9$$

(ii)  $x^3 - 5x + 1 = 0$   
(add  $2x$  to each side)

$$x^3 - 3x + 1 = 2x$$

(add 3 to each side)

$$x^3 - 3x + 4 = 2x + 3$$

So draw the line  $y = 2x + 3$

$$x = -2.3 \text{ or } x = 0.2 \text{ or } x = 2.1$$

### Algebra Revision 3 – Answers

1) (a)  $(x - 6)(x + 2)$       (b)  $(2x + 7)(2x - 7)$       (c)  $(2x - 3)(x - 4)$   
 (d)  $4x(x - 9)$                       (e)  $5x(x - 2)(x - 1)$

2) Use your answers to Q1 to solve the equations:  
 (a)  $x = 6$  or  $x = -2$       (b)  $x = 3.5$  or  $x = -3.5$       (c)  $x = 1.5$  or  $x = 4$   
 (d)  $x = 0$  or  $x = 9$

3) Simplify:  
 (a)  $\frac{5(x-2)-3(x-3)}{(x-3)(x-2)} = \frac{2x-1}{(x-3)(x-2)}$

(b)  $\frac{x(x+2)-3(x-5)}{x(x-5)} = \frac{x^2-x+15}{x(x-5)}$

4)  $3x^2 - 4x - 5 = 0$  so  $x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \times 3 \times -5}}{6} = 2.12$  or  $-0.79$

5)  $x + y = 140$   
 $2x + 3y = 340$                       so  $x = 80$  and  $y = 60$

6)  $(x-3)(x+2) = 84$   
 $x^2 - x - 6 = 84$   
 $x^2 - x - 90 = 0$                       so  $(x - 10)(x + 9) = 0$   
 $x = 10$  (or  $x = -9$  but we can't have negative lengths)

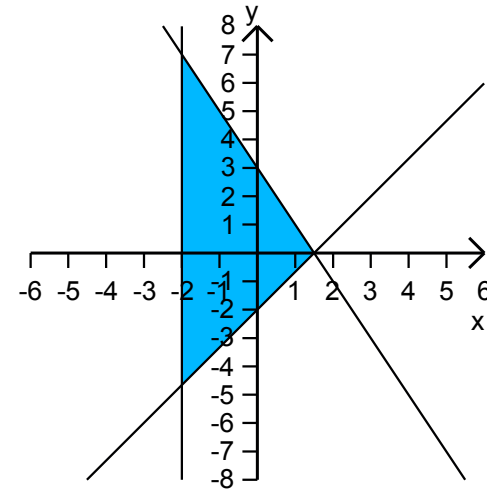
7) (i)  $\sqrt{\frac{4A}{3}} = h$                       (ii)  $h = \frac{p}{R^2}$                       (iii)  $M(a-h) = a+2h$   
 $Ma - Mh = a+2h$   
 $Ma - a = Mh + 2h$   
 $Ma - a = h(M+2)$   
 $\frac{Ma - a}{M+2} = h$

8) (i)  $x^2$       (ii)  $x^4$       (iii)  $x^4$

9) (a)

x	-3	-2	-1	0	1	2	3
y	-6	$-4\frac{2}{3}$	$-3\frac{1}{3}$	-2	$-2\frac{2}{3}$	$2\frac{1}{3}$	2

(bcd)



## Algebra Revision 4 – Answers

### Formulae, negative numbers etc

$$1) x = \frac{\sqrt{144+36}}{17.64} = 0.761(3\text{sf})$$

$$2) \text{(a)} 49 - 16 = 33 \quad \text{(b)} -4 \times 9 = -36 \quad \text{(c)} \frac{7+8}{-3} = -5$$

### Multiplying brackets, solving equations

$$3) \text{(a)} \quad \begin{aligned} 4(3x - 5) - (2x - 10) &= 6(x - 3) \\ 12x - 20 - 2x + 10 &= 6x - 18 \\ 10x - 10 &= 6x - 18 \\ 4x &= -8 \\ x &= -2 \end{aligned}$$

$$\text{(b)} \quad \begin{aligned} \frac{4}{2x+3} &= \frac{10}{8x+3} \\ 4(8x+3) &= 10(2x+3) \\ 32x+12 &= 20x+30 \\ 12x &= 18 \\ x &= 1\frac{1}{2} \end{aligned}$$

$$4) \text{(a)} (2x - 5)(3x + 4y) = 6x^2 + 8xy - 15xy - 20y^2 \\ = 6x^2 - 7xy - 20y^2$$
$$\text{(b)} (4x + 5)(4x - 5) = 16x^2 - 20x + 20x - 25 \\ = 16x^2 - 25$$
$$\text{(c)} (2x - 3)^2 = (2x - 3)(2x - 3) \\ = 4x^2 - 6x - 6x + 9 \\ = 4x^2 - 12x + 9$$

### Simultaneous Equations

$$5) \quad \begin{array}{ll} 4x - 3y = 15 & \text{(A)} \\ 6x - 5y = 24 & \text{(B)} \end{array}$$

$$\text{(A)} \times 3 \quad 12x - 9y = 45 \quad \text{(C)}$$

$$\text{(B)} \times 2 \quad 12x - 10y = 48 \quad \text{(D)}$$

$$\text{(C)} - \text{(D)} \quad y = -3 \quad (\text{because } -9y - (-10y) = -9y + 10y = y)$$

Substituting in (A):  $4x + 9 = 15$

$$4x = 6$$

$$x = 1\frac{1}{2}$$

### Factorising

$$6) \text{(a)} 18x^2y - 12xyz = 6xy(3x - 2z)$$

$$\text{(b)} 4x^2 - 16x = 4x(x - 4)$$

$$\text{(c)} 4x^2 - 16 = 4(x^2 - 4) = 4(x + 2)(x - 2)$$

$$\text{(d)} 25x^2 - 81y^2 = (5x + 9y)(5x - 9y)$$

$$7) \text{(a)} x^2 + 10x + 24 = (x + 6)(x + 4)$$

$$\text{(b)} x^2 + 10x - 24 = (x + 12)(x - 2)$$

$$\text{(c)} x^2 - 10x + 24 = (x - 6)(x - 4)$$

$$\text{(d)} x^2 - 10x - 24 = (x - 12)(x + 2)$$

$$8) \text{(a)} 2x^2 + x - 15$$

[Think: two numbers that multiply to -30 and add to 1 ??

Aha - -6 and 5]

$$= 2x^2 - 6x + 5x - 15$$

$$= 2x(x - 3) + 5(x - 3)$$

$$= (2x + 5)(x - 3)$$

$$\text{(b)} 6x^2 - 11x + 4$$

[Think: two numbers that multiply to 24 and add to -11 ??

Aha - -8 and -3]

$$= 6x^2 - 8x - 3x + 4$$

$$= 2x(3x - 4) - 1(3x - 4)$$

$$= (2x - 1)(3x - 4)$$

## Solving Quadratic Equations

9) (a)  $x^2 = 7x + 18$   
 $x^2 - 7x - 18 = 0$   
 $(x - 9)(x + 2) = 0$   
 $x - 9 = 0$  or  $x + 2 = 0$   
 $x = 9$  or  $x = -2$

(b)  $3x^2 + 12 = 15x$   
 $3x^2 - 15x + 12 = 0$   
(either:  $\div$  both sides by 3)  
 $x^2 - 5x + 4 = 0$   
 $(x - 4)(x - 1) = 0$   
 $x = 4$  or  $x = 1$

(or think: two numbers which multiply to 36 and add to  $-15$   
Numbers are  $-12$  and  $-3$ )

$$3x^2 - 12x - 3x + 12 = 0$$
$$3x(x - 4) - 3(x - 4) = 0$$
$$(3x - 3)(x - 4) = 0$$
$$3x - 3 = 0 \text{ or } x - 4 = 0$$
$$x = 1 \text{ or } x = 4$$

10)  $5x^2 = 3x + 13$   
 $5x^2 - 3x - 13 = 0$   
 $a = 5, b = -3, c = -13$ 

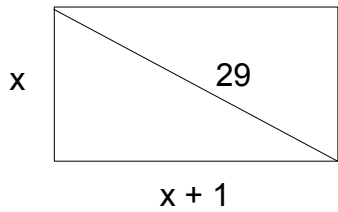
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$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \times 5 \times -13}}{2 \times 5}$$
$$x = \frac{3 \pm \sqrt{9 + 260}}{10}$$
$$x = 1.94 \text{ or } -1.31 \text{ (3sf)}$$

## Problems solved by equations

11) Tom's mother is  $x$  years old  
Tom's father is  $80 - x$   
Tom's grandmother is  $3x$       Tom's grandmother is also  $2(80 - x)$   
So  $3x = 2(80 - x)$   
 $3x = 160 - 2x$   
 $5x = 160$   
 $x = 32$   
So Tom's mother is 32, father is 48 and grandmother is 96

12)

By Pythagoras,  $x^2 + (x + 1)^2 = 29^2$ 

$$x^2 + x^2 + 2x + 1 = 841$$

$$2x^2 + 2x - 840 = 0$$

$$(\div \text{ both sides by 2}) \quad x^2 + x - 420 = 0$$

$$(x + 21)(x - 20) = 0$$

$$x + 21 = 0 \text{ or } x - 20 = 0$$

$$x = -21 \text{ (not sensible) or } x = \mathbf{20}$$

13) For £6.60 I can either buy 3 Chocos and 5 Twizzlers, or 4 Chocos and 3 Twizzlers. Form two equations, and solve them simultaneously to find the cost of each type of sweet.

Let  $c$  be the price of a choco and  $t$  be the price of a twizzler

So  $3c + 5t = 660$  (A)

$4c + 3t = 660$  (B)

(A)  $\times 3$   $9c + 15t = 1980$  (C)

(B)  $\times 5$   $20c + 15t = 3300$  (D)

(D)  $-$  (C)  $11c = 1320$

$c = 120$

Sub in (A)  $360 + 5t = 660$

$5t = 300$

$t = 60$

A Choco costs £1.20 and a Twizzler costs 60p