# Calculus Practice

1) In each case, find the gradient at the given point on the curve:

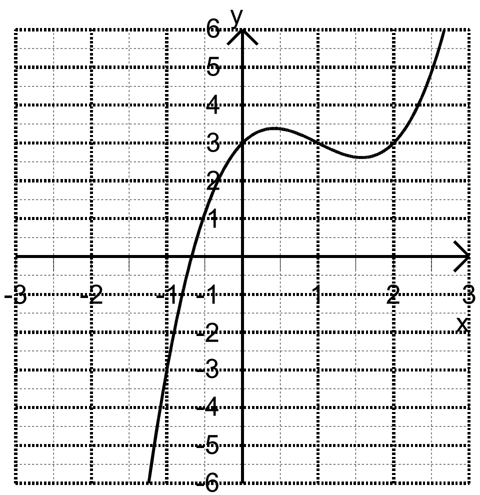
(a) $y = x^3$, at the point (3, 27)
(b) $y = x^5$, at the point (-1, -1)
(c) $y = 4x^2$, at the point (1.5, 9)
(d) $y = x^3 - x^2$, at the point (2, 4)

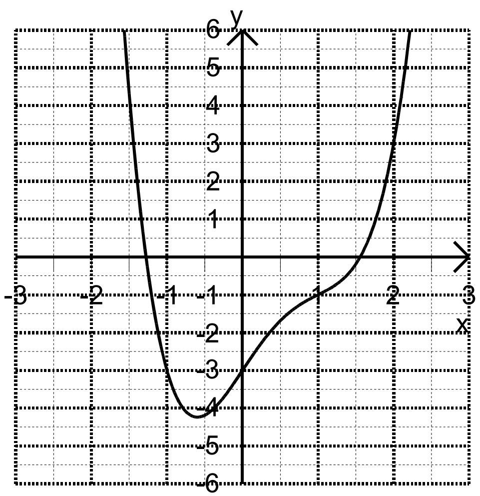
2) Find $\frac{dy}{dx}$ for each of the following:

(a) $y = 3x^6$ (b) $y = x^7 + x^4$ (c) $y = 4x^5 - 3x^3$ (d) $y = 5x$ (e) $y = 7$
(f) $y = x^4 + x^3 - 3x - 9$ (g) $y = x^3 - 5x^2 + 7x + 3$ (h) $y = 2x^3 - 6x^2$

3) (a) The graph on the left is of the equation $y = x^3 - 3x^2 + 2x + 1$. By drawing tangents, find the gradient of the graph at the points (i) (-1, -3) and (ii) (2, 3). Now find these gradients by differentiation. How accurate were your tangents?

(b) The graph on the right is of the equation $y = x^4 - 2x^3 + 3x - 3$. Repeat part (a) for this graph.




4) In each case, find the gradient at the given point on the curve:

(a) $y = x^{-3}$, at the point (2, 0.125)
(b) $y = \frac{1}{x}$, at the point (4, 0.25)
(c) $y = \frac{4}{x^2}$, at the point (–2, 1)
(d) $y = \frac{2}{x^3} - 4x^3$, at the point (–1, 2)

5) Find $\frac{dy}{dx}$ for each of the following:

(a) $y=2x^3-3x^2+\frac{5}{x}-3$ (b) $y=7x^4-2x^3+6x-\frac{1}{x^2}$ (c) $y=\frac{2}{x^3}-\frac{5}{x^2}+\frac{3}{x}$

6) For each of the following equations,

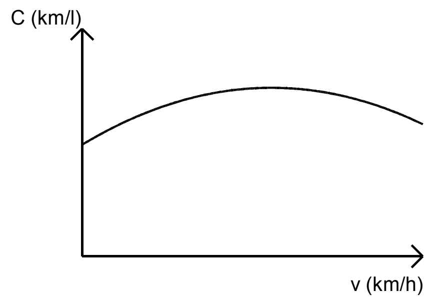
(i) Find the turning point(s)
(ii) sketch the graph, showing turning point(s) and y-intercept

(a) $y = x^2 - 6x + 7$ (b) $y = 8 + 4x - x^2$ (c) $y = x^3 - 12x + 5$
(d) $y = 7 - 3x - x^3$ (e) $y = x^3 - 3x^2 + 4$

7) The UK population (P) of a certain reptile t years after records started being kept is given by the formula $P = t^3 + 2t^2 + 100$. Find the rate at which the population is increasing 5 years after records started being kept.

8) The volume (V kilolitres) of water in a reservoir t months after the start of a year is given by the formula $V = 100t^2 - 1600t + 10000$.
Find the rate at which the volume is changing:
(a) in May (ie when t = 5)
(b) in October

9) The graph shows the petrol usage (C) of a car (in km per litre) at different speeds.
The formula for C is $C = 7 + 0.16v - 0.001v^2$




By finding the coordinates of the maximum point of this graph, find the most economical speed at which to drive the car, and the number of kilometres which the car will travel per litre of fuel at this speed.

10) A stone is thrown straight up in the air. The formula for the height (s) of the stone after t seconds is $s = 40t - 5t^2$.
(a) Find the formula for the velocity v of the stone.
(b) Find the velocity after (i) 2 seconds (ii) 5 seconds.
(c) What does the sign of the answer to (b)(ii) indicate?
(d) Find the time at which the velocity of the stone is 0. What point on its journey is this?
(e) Use your answer to (d) to find the maximum height reached by the stone.
(f) Find the acceleration of the stone.

11) An object moves so that its displacement (s) from its starting point after t seconds is given by $s = t^3 - 48t$.
(a) Find the formula for the velocity v of the stone.
(b) Find the velocity after 5 seconds
(c) Find the time when the velocity is 0.
(d) Find the formula for the acceleration of the object.
(e) Find the acceleration after 5 seconds.

12) A firm producing pizzas estimates that the cost ($C$ pence) of producing a pizza is related to the number of pizzas produced in a week ($p$ thousand) by the formula $C = 100 - 0.5p + 0.01p^2$. Find the number of pizzas which should be produced in a week to minimise the cost per pizza.

13) An open box is to be produced by removing a square of side $x$ cm from each corner of a square piece of card with sides of 30cm, and folding up the sides.
(a) Show that the volume of the box is given by $V = 4x^3 - 120x^2 + 900x$
(b) Hence find the value of $x$ which maximises the volume of the box.
(c) What is the maximum volume of the box?