SETS

Sets can be described in words :—

    e.g. $A = \{$ factors of $12\}$

or as a list

    e.g $B = \{1, 3, 5, 7, 9\}$

           $C = \{2, 3\}$

There are various symbols used with sets :—

        $\in$ 'is a member of'

  e.g. $7 \in B$

        $\subset$ 'is a subset of'

  e.g. $C \subset A$

    (because $A = \{1, 2, 3, 4, 6, 12\}$

                   $\uparrow C$

We can put a line through these symbols to negate them.

    e.g $6 \notin B$

        $C \not\subset B$

The symbol $\cap$ means the INTERSECTION of 2 sets ie, the items which are in both sets.

    e.g. $A \cap B = \{1, 3\}$

          $B \cap C = \{3\}$

The symbol $\cup$ means the UNION of two sets ie, the items which are in one set or the other or both

e.g $\quad A \cup B = \{1, 2, 3, 4, 5, 6, 7, 9, 12\}$

(only include each item once)

The symbol $n(A)$ means the number of items in set A.

So
$$n(A) = 6$$
$$n(B) = 5$$
$$n(c) = 2$$

The symbol $\mathcal{E}$ (which is a fancy E) means the 'Universal Set' (E for 'Everything') ie, everything which is included in the question.

We will say that $\mathcal{E} = \{1, 2, 3, 4, 5, 6, 7, \ldots 12\}$

The symbol A' means all items which are in $\mathcal{E}$ but are not in A (ie, the COMPLEMENT of A)

So
$$A' = \{5, 7, 8, 9, 10, 11\}$$
$$B' = \{2, 4, 6, 8, 10, 11, 12\}$$

The symbols $\{\ \}$ or $\emptyset$ are alternative ways of indicating an EMPTY SET ie a set with no members.

e.g if $P = \{1, 3\}$ and $Q = \{2, 4\}$
then $P \cap Q = \{\ \}$
                        or $\emptyset$

Venn Diagrams are a way of illustrating sets
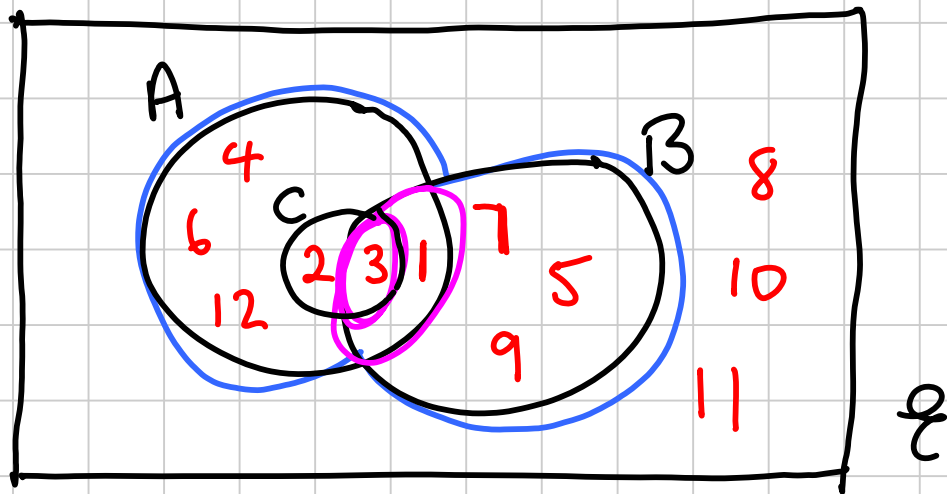
e.g for sets $\mathcal{E} = \{1, 2, 3, \ldots 12\}$
              $A = \{1, 2, 3, 4, 6, 12\}$
              $B = \{1, 3, 5, 7, 9\}$
              $C = \{2, 3\}$

we have a rectangle for $\mathcal{E}$ and a circle for
each of A, B and C.
Since $C \subset A$, the circle for C is inside A
Since $A \cap B \neq \emptyset$, their circles overlap
       $B \cap C \neq \emptyset$, their circles overlap

Sometimes sets are expressed using words rather than lists.

Example

$\xi = \{$ students in Y11 Div 3 $\}$

$A = \{$ students in a school sports team $\}$

$B = \{$ students who come by bus $\}$

$C = \{$ student who study drama $\}$

Describe the following sets :-

(a) $A \cap B = \{$ students in Div 3 who are in a sports team and come by bus $\}$

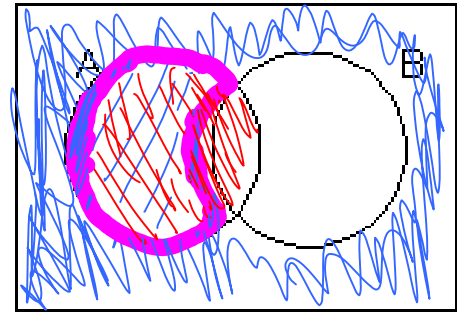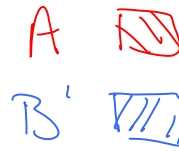$A \cup C = \{$ students in Div 3 who are in a sports team or study drama (or both) $\}$

$B \cap C' = \{$ students in Div 3 who come by bus but don't study drama $\}$

$B \cap (A \cup C) = \{$ students in Div 3 who come by bus and either are in a sports team or study drama $\}$

Note that $\quad \cap \implies$ AND $\left.\phantom{\begin{matrix}a\\b\end{matrix}}\right\}$
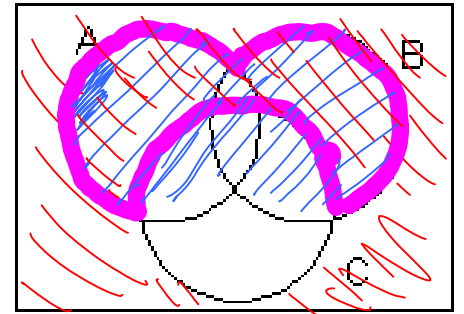$\qquad \qquad \cup \implies$ OR

Shading regions on Venn diagrams

1) On the diagram on the right, shade the region A ∩ B'

A ▱
B' ▨
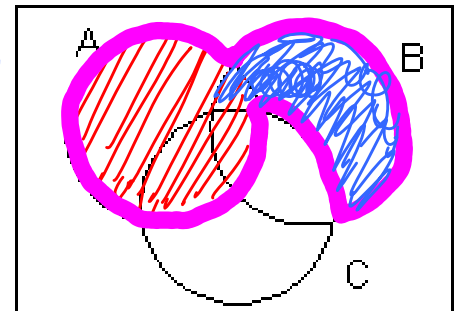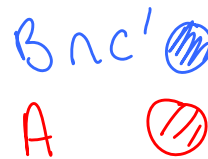Answer ⬭

2) On the diagram on the right, shade the region (A ∪ B) ∩ C'

A ∪ B ▱
C' ▨
Answer ⬭

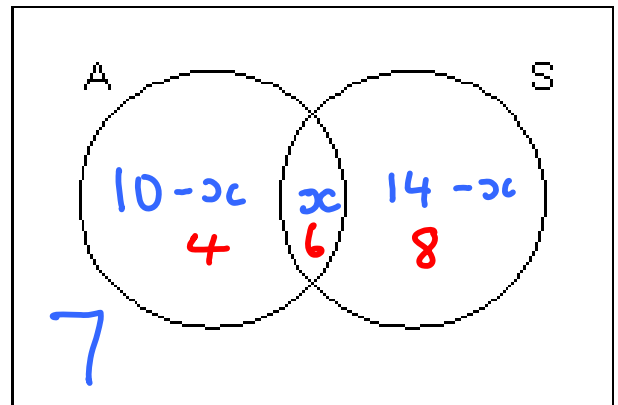3) On the diagram on the right, shade the region A ∪ (B ∩ C')

B ∩ C' ⊘
A ⊘
Answer ⬭

Problems involving numbers in sets

1) Out of 25 students in a form, 10 study Art, and 14 study Spanish. 7 students study neither. How many study Spanish but not Art?

$10 - x + x + 14 - x + 7 = 25$

$$31 - x = 25$$
$$x = 6$$

<u>8</u> study Spanish but not Art

A Venn diagram with two overlapping circles labelled A and S:
- A only: $10 - x$, 4
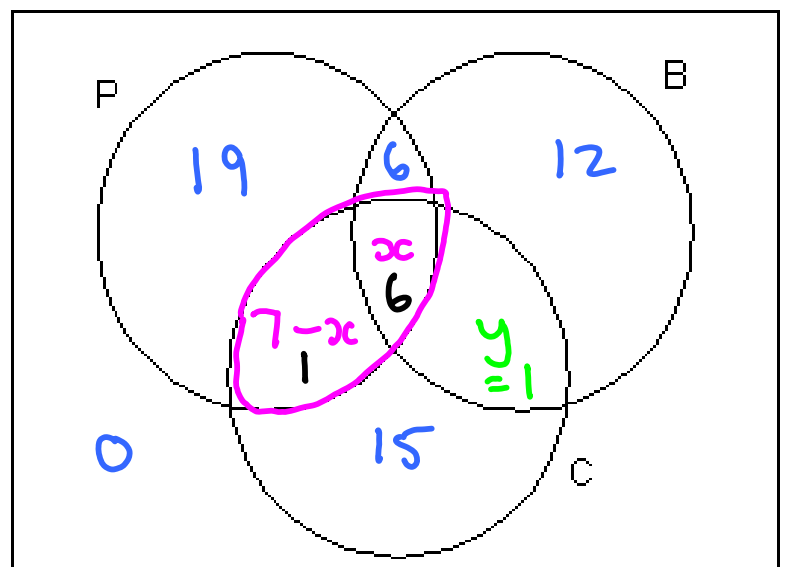- intersection: x, 6
- S only: $14 - x$, 8
- outside: 7

2) All students in a year study at least one science. 19 study Physics only, 12 Biology only and 15 Chemistry only. 6 study Physics and Biology only. 7 study Physics and Chemistry, 23 study Chemistry and 25 study Biology. [Hint: the word "only" does not appear in the previous sentence!] How many students study Physics and Chemistry but not Biology?

Chemistry circle:

$15 + 7 - x + x + y = 23$
$$22 + y = 23$$
$$y = 1$$

Biology circle:

$12 + 6 + x + 1 = 25$
$$19 + x = 25$$
$$x = 6$$

A Venn diagram with three overlapping circles labelled P, B and C:
- P only: 19
- P and B only: 6
- B only: 12
- P and C only: $7 - x$, 1
- centre: x, 6
- B and C only: $y = 1$
- C only: 15
- outside: 0

<u>1</u> student studies P and C but not B