Vectors – Answers

Vectors in component form

1) (a) $\begin{pmatrix} 11 \\ -2 \end{pmatrix}$ (b) $\begin{pmatrix} 3 \\ -6 \end{pmatrix}$ (d) $\begin{pmatrix} 17 \\ 0 \end{pmatrix}$ (e) $\begin{pmatrix} 21 \\ -12 \end{pmatrix}$

2) (a) $\begin{pmatrix} -2 \\ 4 \end{pmatrix}$ (b) $\begin{pmatrix} 36 \\ -2 \end{pmatrix}$ (c) $\begin{pmatrix} -7 \\ 4 \end{pmatrix}$ (d) $\begin{pmatrix} -5 \\ 10 \end{pmatrix}$

3) $|\mathbf{a}| = 5$ $|\mathbf{b}| = 10$ $|\mathbf{c}| = 5$

Vector Questions

1) (a) $2\mathbf{v}$ (b) $2\mathbf{w}$ (c) $\mathbf{w} - \mathbf{v}$ (d) $2\mathbf{w} - 2\mathbf{v}$
BC is parallel to DE and twice as long.

2) $\mathbf{OX} = \frac{2}{3}\mathbf{a}$ $\mathbf{OY} = \frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$ $\mathbf{OZ} = 2\mathbf{b}$ $\mathbf{XY} = \frac{1}{2}\mathbf{b} - \frac{1}{6}\mathbf{a}$ $\mathbf{YZ} = \frac{3}{2}\mathbf{b} - \frac{1}{2}\mathbf{a}$
X, Y and Z are collinear (ie they lie in a straight line), and the length of YZ is 3 times the length of XY.

3) (a) $\sqrt{29}$ (b) (6, 5)   (c) $\begin{pmatrix} -4 \\ 1 \end{pmatrix}$, $\sqrt{17}$ (d) a parallelogram

4) (a) $\mathbf{b}$ (b) $\frac{1}{2}\mathbf{b}$ (c) $\mathbf{a} + \frac{1}{4}\mathbf{b}$ (d) $\frac{2}{3}\mathbf{a} + \frac{1}{6}\mathbf{b}$
    (e) $\frac{1}{6}\mathbf{b} - \frac{1}{3}\mathbf{a}$ (f) $\frac{1}{3}\mathbf{b} - \frac{2}{3}\mathbf{a}$
    (g) $\mathbf{FE} = 2\mathbf{AF}$, so FE points in the same direction as AF but is twice as long.
    (h) AF:FE = 1:2

5) (a) $\mathbf{b} - \mathbf{a}$ (b) $\frac{2}{3}\mathbf{a} - \frac{1}{6}\mathbf{b}$ (c) $4\mathbf{a} - \mathbf{b}$
(d) BR is parallel to PQ and 6 times as long

6) (a) $3\mathbf{a} - \mathbf{b}$ (b) $\frac{3}{4}\mathbf{a} - \frac{1}{4}\mathbf{b}$ (c) $\mathbf{b} - \mathbf{a}$ (d) $\frac{3}{4}\mathbf{b} - \frac{1}{4}\mathbf{a}$
    (e) $\frac{9}{4}\mathbf{b} - \frac{3}{4}\mathbf{a}$ (f) $3\mathbf{b} - 3\mathbf{a}$
    (g) CF is parallel to AB and 3 times as long.
    (h) A, E and F are collinear, and EF is 3 times as long as AE.

7) $\mathbf{AM} = \mathbf{p} + \mathbf{q}$ $\mathbf{AC} = \mathbf{p} + 2\mathbf{q}$ $\mathbf{NC} = \frac{1}{2}\mathbf{p} + \mathbf{q}$ $\mathbf{NB} = \frac{1}{2}\mathbf{p} - \mathbf{q}$
    $\mathbf{DB} = \frac{2}{3}\mathbf{p} - \frac{4}{3}\mathbf{q}$ $\mathbf{DC} = \frac{2}{3}\mathbf{p} + \frac{2}{3}\mathbf{q}$
    AM and DC are parallel, and AM:DC = 3:2

Vector equation of a line
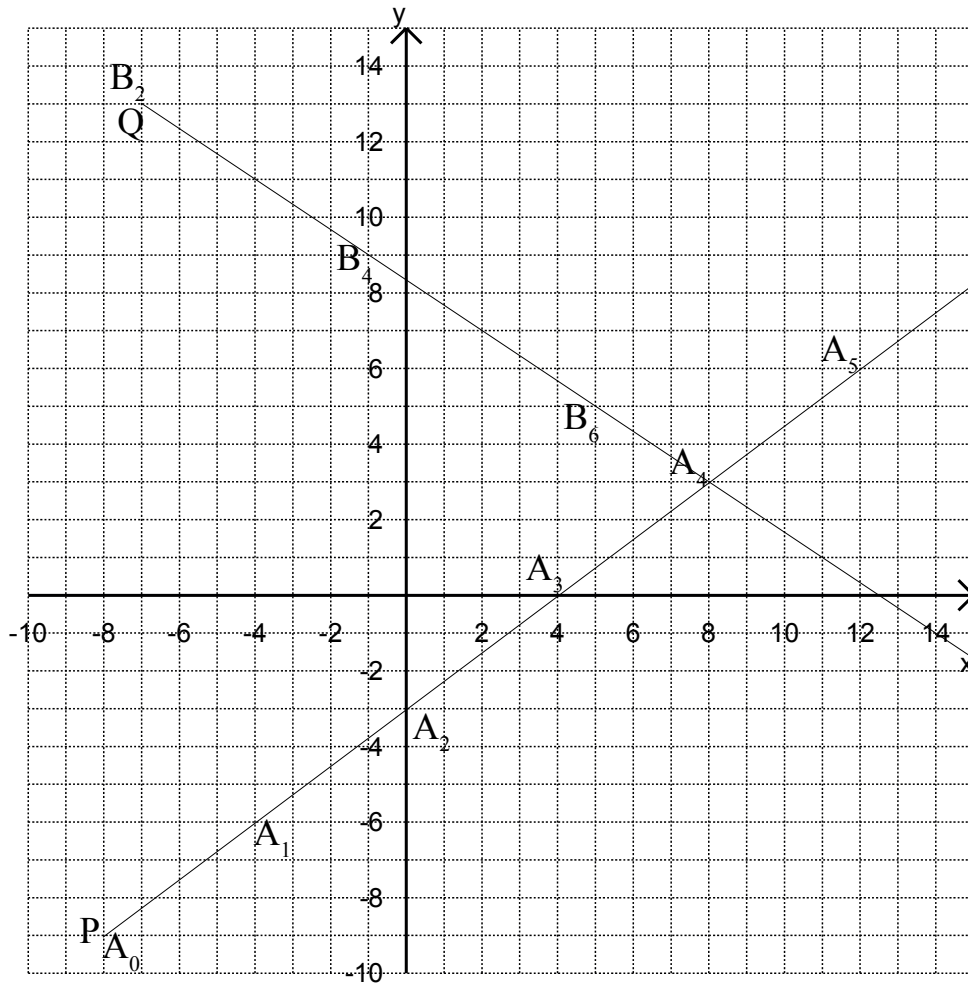
(c) 5 km/h

(e) 3.6 km/h

(f) Paths cross at (8, 3). The "Anna" arrives here 4 hours after leaving port. At this time, the "Betty" is at the point (-1, 9).

(g) The "Betty" arrives at the crossing point at $t = 7$, ie 5 hours after leaving port Q. At this time the "Anna" is at (20, 12)

(h) At $t = 3$, the ships are 13.6 km apart

(i) The "Anna" is sailing on bearing 053.1°
    The "Betty" is sailing on bearing 123.7°