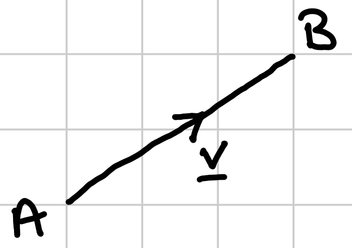
VECTORS

Note Title 29/09/2008

A vector has both:

- Magnitude (size, length)
- direction

A vector can be represented as



$\overrightarrow{AB}$ (geometrical)

or $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ (numerical - provided we have coordinates to use)

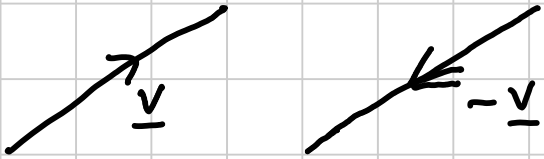
or $\underline{v}$ (algebraic - underlined to show it is a vector - in textbooks, bold type is used instead of underlining)

<u>Equal Vectors</u> If two vectors have the same length <u>and</u> direction they are equal even if they start at different places.

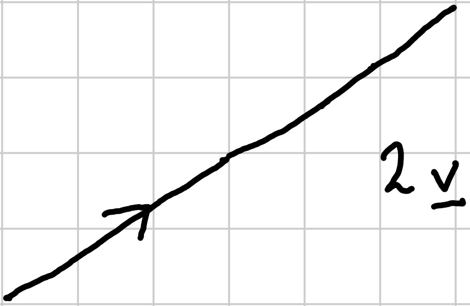
$\underline{v} \neq \underline{w}$ because although they are the same length, they are in different directions

$\underline{v} = \underline{x}$

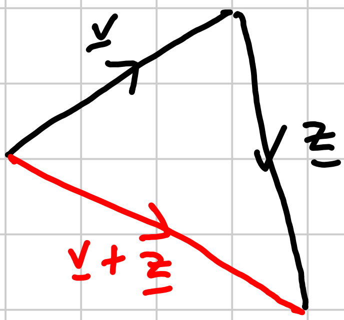
The Negative of a Vector has the same length but in the opposite direction.



A multiple of a Vector has the same direction but a different length



Adding Vectors is done by placing them 'nose-to-tail' and drawing a single vector from the start to the finish.



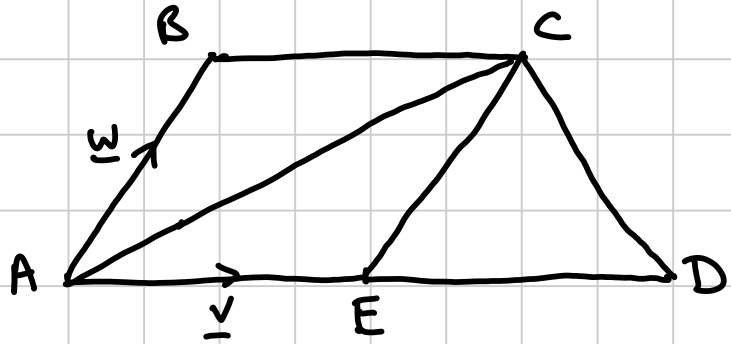
[Numerically:

$$\begin{pmatrix}3\\2\end{pmatrix} + \begin{pmatrix}1\\-4\end{pmatrix} = \begin{pmatrix}4\\-2\end{pmatrix}$$

]

Examples

(1)



ABCE is a parallelogram. $\overrightarrow{AE} = \underline{v}$, $\overrightarrow{AB} = \underline{w}$.
Express the following vectors in terms of $\underline{v}$ and/or $\underline{w}$.

(a) $\overrightarrow{BC} = \underline{v}$

(b) $\overrightarrow{BA} = -\underline{w}$

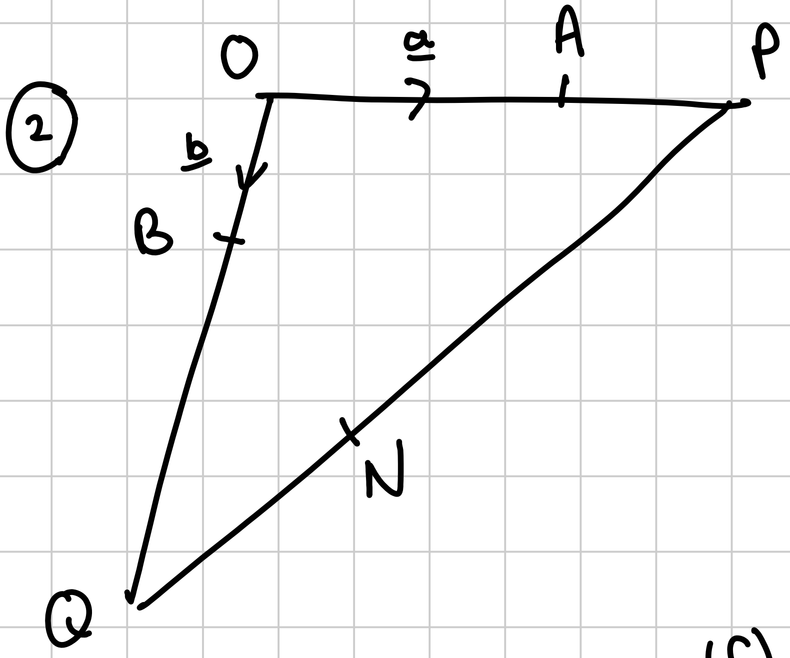
(c) $\overrightarrow{AD} = 2\underline{v}$

(d) $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$
$= \underline{w} + \underline{v}$

(e) $\overrightarrow{BE} = \overrightarrow{BC} + \overrightarrow{CE}$
$= \underline{v} + (-\underline{w})$
$= \underline{v} - \underline{w}$

(f) $\overrightarrow{CD} = \overrightarrow{CE} + \overrightarrow{ED}$
$= -\underline{w} + \underline{v}$ (or $\underline{v} - \underline{w}$)

(2)



In this diagram,
A divides OP in the ratio 2 : 1
B divides OQ in the ratio 1 : 3
Express the following vectors in terms of $\underline{a}$ and $\underline{b}$

(a) $\overrightarrow{AP} = \frac{1}{2}\underline{a}$

(b) $\overrightarrow{AB} = -\underline{a} + \underline{b}$

(c) $\overrightarrow{OQ} = 4\underline{b}$

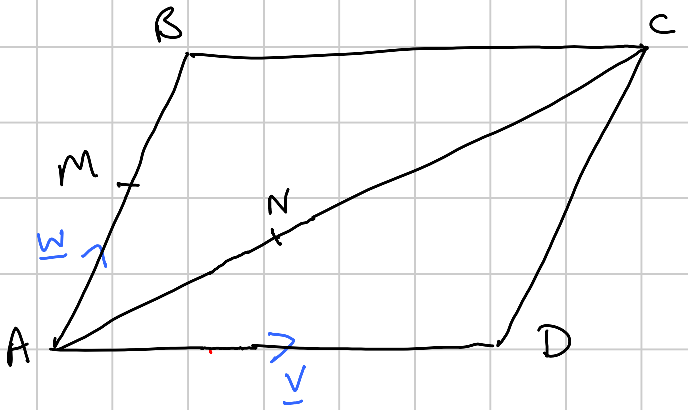
(d) $\overrightarrow{PO} = -1\frac{1}{2}\underline{a}$

(e) $\overrightarrow{PQ} = \overrightarrow{PO} + \overrightarrow{OQ}$
$= -1\frac{1}{2}\underline{a} + 4\underline{b}$

(f) $\overrightarrow{PN} = \frac{2}{3}\overrightarrow{PQ}$
$= \frac{2}{3}(-1\frac{1}{2}\underline{a} + \frac{4}{1}\underline{b})$
$= -1\underline{a} + \frac{8}{3}\underline{b}$
(or $-1\underline{a} + 2\frac{2}{3}\underline{b}$)

(g) $\overrightarrow{ON} = \overrightarrow{OP} + \overrightarrow{PN}$
$= 1\frac{1}{2}\underline{a} + -1\underline{a} + \frac{8}{3}\underline{b}$
$= \frac{1}{2}\underline{a} + \frac{8}{3}\underline{b}$

Using Vectors to prove Geometric Facts



ABCD is a parallelogram
M is the midpoint of AB
NC = 2AN
AD = $\underline{v}$ and AM = $\underline{w}$

Express in terms of $\underline{v}$ and/or $\underline{w}$:—

(a) $\overrightarrow{AC} = \overrightarrow{AD} + \overrightarrow{DC}$
$= \underline{v} + 2\underline{w}$

(b) $\overrightarrow{AN} = \frac{1}{3}\overrightarrow{AC}$

$$= \tfrac{1}{3}(\underline{v} + 2\underline{w})$$
$$= \tfrac{1}{3}\underline{v} + \tfrac{2}{3}\underline{w}$$

(c) $\overrightarrow{MN} = \overrightarrow{MA} + \overrightarrow{AN}$

$$= -\underline{w} + \tfrac{1}{3}\underline{v} + \tfrac{2}{3}\underline{w}$$
$$= \tfrac{1}{3}\underline{v} - \tfrac{1}{3}\underline{w} \qquad \left(-\tfrac{1}{3}\underline{w} + \tfrac{1}{3}\underline{v} \text{ is the same}\right)$$

(d) $\overrightarrow{ND} = \overrightarrow{NA} + \overrightarrow{AD}$

$$= -\tfrac{1}{3}\underline{v} - \tfrac{2}{3}\underline{w} + \underline{v}$$
$$= \tfrac{2}{3}\underline{v} - \tfrac{2}{3}\underline{w}$$

(e) Using your answers to (c) and (d), state two facts about M, N and D.

$$\overrightarrow{ND} = 2\,\overrightarrow{MN}$$

We deduce that :-

- the distance ND = 2 × the distance MN
- $\overrightarrow{ND}$ is in the same direction as $\overrightarrow{MN}$

  ie, M, N and D lie in a straight line

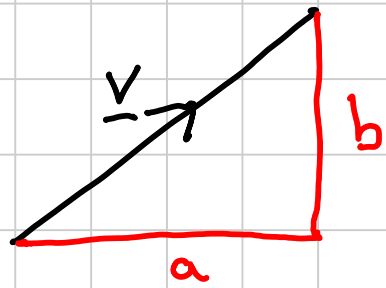
If $\overrightarrow{AB} = k \times \overrightarrow{CD}$, we can deduce two things

- one about the LENGTHS
- one about the DIRECTIONS
  - either that points are in a line, or that lines are parallel

(k could be any number)

Magnitude of a Vector

If a vector is given in the form $\begin{pmatrix} a \\ b \end{pmatrix}$



then we can find the magnitude (ie length) of the vector by using Pythagoras:

$$\text{magnitude of } \underline{v} = \sqrt{a^2 + b^2}$$

The magnitude can be written as $|\underline{v}|$.

<u>Example</u> If $\underline{v} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$, find $|\underline{v}|$

$$|\underline{v}| = \sqrt{(-3)^2 + 2^2}$$
$$= \sqrt{9 + 4}$$
$$= \sqrt{13}$$

(= 3.6 units to 1 dp)