SOLID SHAPES — VOLUME AND SURFACE AREA
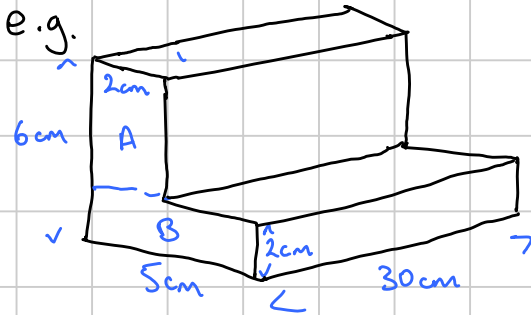
Prism

A prism is a solid shape with a CONSTANT CROSS-SECTION

(ie, if we cut through it anywhere the cut looks the same)

e.g.

$$\boxed{\text{Volume} = \text{area of cross-section} \times \text{length}}$$

Area of cross-section

$$= \text{Area A} + \text{Area B}$$
$$= (4 \times 2) + (5 + 2)$$
$$= 18 \, cm^2$$

$$\text{Volume} = 18 \times 30$$
$$= 540 \, cm^3$$

Cylinder

A cylinder is a prism with a circular cross-section

So area of cylinder $= \pi r^2 \times$ length or height

$$= \pi r^2 h$$

Also, curved surface area $= 2\pi r h$

Example

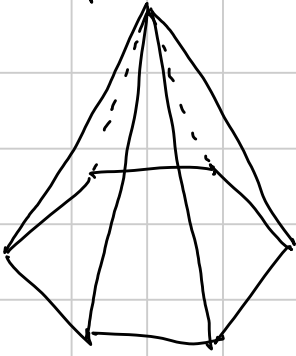Find the volume and surface area of a tin with diameter 10cm and height 12cm

$$\text{Volume} = \pi \times 5^2 \times 12 = \underline{942 \, cm^3}$$

$$\text{Curved surface area} = 2 \times \pi \times 5 \times 12 = 377 \, cm^2$$

$$\text{Area of top} = \pi \times 5^2 = 78.5 \, cm^2$$

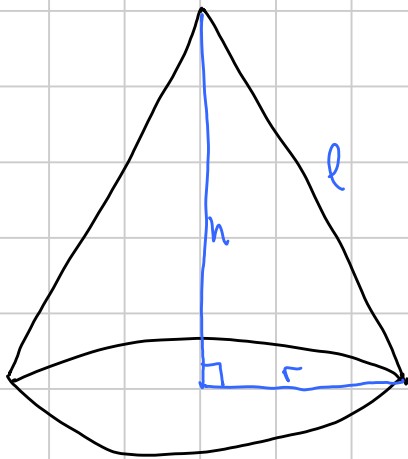$$\text{Total surface area} = 377 + (2 \times 78.5) = \underline{534 \, cm^2}$$

Pyramids

A pyramid can have many different shapes of base.

$$\text{Volume of pyramid} = \frac{1}{3} \times \text{area of base} \times \text{perp height}$$

Cone

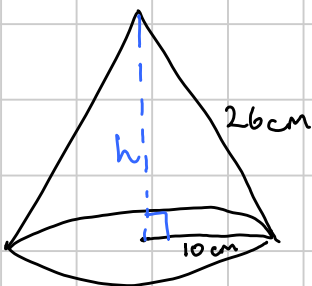A cone is a pyramid with a circular base

$$\text{Volume} = \frac{1}{3}\pi r^2 h$$

$$\text{Curved surface area} = \pi r l$$
(where l = the slant height of the cone)

Also, by Pythagoras, $h^2 + r^2 = l^2$

Example

(a) Find the **total** surface area (including the base)

$$\text{Curved S.A} = \pi \times 10 \times 26 = 816.81\,cm^2$$
$$\text{Base} \quad = \pi \times 10^2 \quad = 314.16\,cm^2$$
$$\text{Total SA} \quad \underline{1230.97\,cm^2}$$

(b) Find the volume

First find h:

$$h^2 + 10^2 = 26^2$$
$$(-10^2) \qquad (-10^2)$$
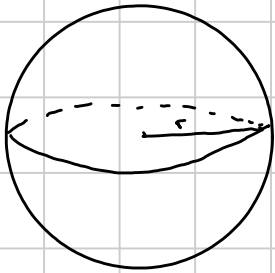
$$h^2 \qquad\qquad = 26^2 - 10^2$$

$$h^2 = 676 - 100$$
$$h^2 = 576$$
$$h = \sqrt{576} = 24 \text{ cm}$$

$$\text{Volume} = \frac{1}{3} \times \pi \times 10^2 \times 24$$
$$= \underline{\underline{2513}} \text{ cm}^3$$

Sphere

$$\text{Volume} = \frac{4}{3}\pi r^3$$

$$\text{Surface Area} = 4\pi r^2$$

Example The volume of a sphere is 1000 cm^3
What is its surface area?

First find r

$$\frac{4}{3}\pi r^3 = 1000$$

$(\times 3)$ $(\times 3)$

$$4\pi r^3 = 3000$$

$(\div 4)$ $(\div 4)$

$$\pi r^3 = 750$$

$(\div \pi)$ $(\div \pi)$

$$r^3 = 238.73\ldots$$

(cube root) (cube root)

$$r = \sqrt[3]{238.73}$$

$$= 6.2034$$

$$\text{Surface area} = 4\pi \times 6.2034^2$$

$$= \underline{\underline{483.6}} \text{ cm}^2$$