Combining Probabilities

Suppose we have a bag containing 24 counters: 8 red counters numbered 1 to 8, 8 blue counters numbered 1 to 8, 8 green counters numbered 1 to 8.

A counter is taken from the bag at random.

Example 1

p(Red) = $\frac{8}{24} = \frac{1}{3}$ p(Odd number) = $\frac{12}{24} = \frac{1}{2}$

p(Red AND an Odd number) = p(Red1, Red3, Red5 or Red7) = $\frac{4}{24} = \frac{1}{6}$

Note that: $\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$

This example illustrates the rule that:

If we know p(X) and p(Y), we can find p(X AND Y) by MULTIPLYING suitable probabilities

Example 2

p(Odd number) = $\frac{12}{24} = \frac{1}{2}$ p(8) = $\frac{3}{24} = \frac{1}{8}$

p(Odd number OR 8) = $\frac{15}{24} = \frac{5}{8}$

Note that: $\frac{12}{24} + \frac{3}{24} = \frac{15}{24}$ (or $\frac{1}{2} + \frac{1}{8} = \frac{5}{8}$)

Example 3

p(Red) = $\frac{8}{24} = \frac{1}{3}$ p(8) = $\frac{3}{24} = \frac{1}{8}$

p(Red OR 8) = $\frac{10}{24} = \frac{5}{12}$

Note that: $\frac{8}{24} + \frac{3}{24} \neq \frac{10}{24}$ (one bead is both Red and an 8)

These two examples illustrate the rule that:

If we know p(X) and p(Y), we can find p(X OR Y) by ADDING p(X) and p(Y), PROVIDED that X and Y are MUTUALLY EXCLUSIVE ie they cannot both happen

Example 4

Ann and Briony are both going to take their driving test. The probability that Ann passes is $\frac{2}{3}$, and that Briony passes is $\frac{3}{4}$

(a) Find the probability that they both pass

$$p(\text{A passes AND B passes}) = \frac{2}{3} \times \frac{3}{4}$$
$$= \frac{6}{12}$$
$$= \underline{\underline{\frac{1}{2}}}$$

(b) Can we find the probability that Ann OR Briony passes by adding $\frac{2}{3}$ and $\frac{3}{4}$?

No – the events are NOT mutually exclusive
(they could both pass)

Example 5

There are 4 candidates in a mock election at school, including Ann and Briony. Only one candidate will be elected. p(Ann wins) = 0.4, and p(Briony wins) = 0.3

What is the probability that Ann OR Briony is elected?

Ann winning and Briony winning are MUTUALLY EXCLUSIVE events (they can't both happen).

$$p(\text{Ann or Briony is elected}) = 0.4 + 0.3$$
$$= \underline{\underline{0.7}}$$

Tree diagrams

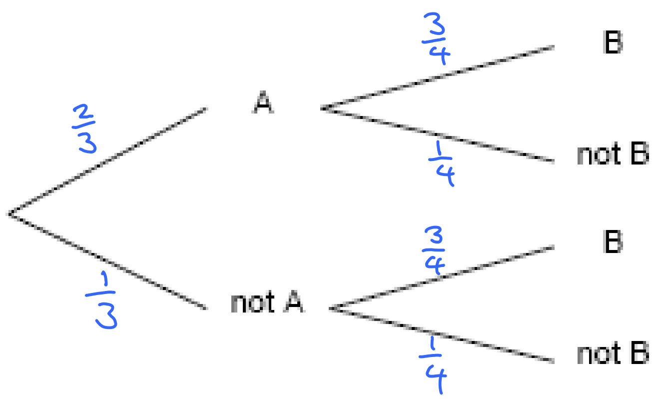
These are a way of showing all possibilities in a systematic way.

Example 1

Ann and Briony are both going to take their driving test. The probability that Ann passes is $\frac{2}{3}$, and that Briony passes is $\frac{3}{4}$

Find the probability that at least one of them passes.

[Let A mean "Ann passes" and B mean "Briony passes"]



p(A AND B) = $\frac{2}{3} \times \frac{3}{4} = \frac{6}{12} = \frac{1}{2}$ *

p(A AND not B) = $\frac{2}{3} \times \frac{1}{4} = \frac{2}{12} = \frac{1}{6}$ *

p(not A AND B) = $\frac{1}{3} \times \frac{3}{4} = \frac{3}{12} = \frac{1}{4}$ *

p(not A AND not B) = $\frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$

At least one girl passes on each of the top three branches. So:

p(at least one passes) = $\frac{6}{12} + \frac{2}{12} + \frac{3}{12} = \underline{\underline{\frac{11}{12}}}$

This illustrates the method for tree diagrams:

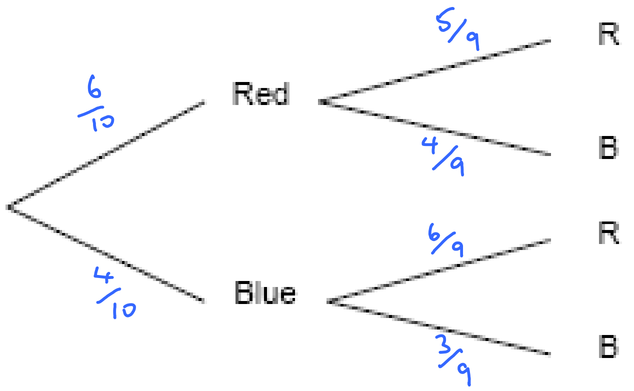
- **Multiply** the probabilities along the branches to get the "AND" probability at the end of the branch
- Select the required branches and **add** the probabilities at the end of those branches to get the answer

[We really only need to work out the probability at the end of the required branches.]

Example 2

A bag contains 6 red beads and 4 blue beads. Two beads are taken from the bag. Find the probability of getting two beads of the same colour

[Note that the probabilities for the second bead will **change** depending on what colour the first bead was.]



p(R then R) = $\frac{6}{10} \times \frac{5}{9} = \frac{30}{90}$

p(B then B) = $\frac{4}{10} \times \frac{3}{9} = \frac{12}{90}$

p(two beads of the same colour) = $\frac{30}{90} + \frac{12}{90}$

$= \frac{42}{90}$ or $\underline{\frac{7}{15}}$