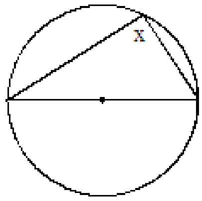
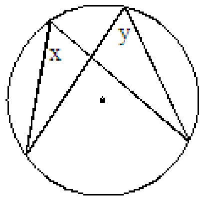
Circle Theorems

1) An angle in a semicircle is a right angle.



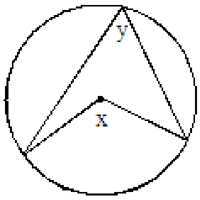
x = 90°

2) Angles in the same segment are equal.

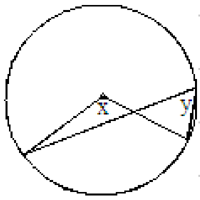


x = y

3) The angle at the centre is twice the angle at the circumference.

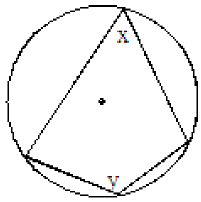


x = 2y



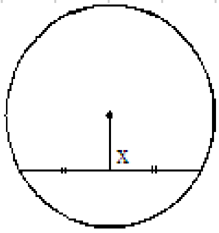
x = 2y

4) Opposite angles of a cyclic quadrilateral add up to 180°.
(A **cyclic** quadrilateral is one with all four vertices on the circumference of a circle.)



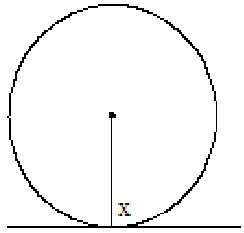
x + y =180°

5) The line from the centre of a circle to the midpoint of a chord is perpendicular to the chord.



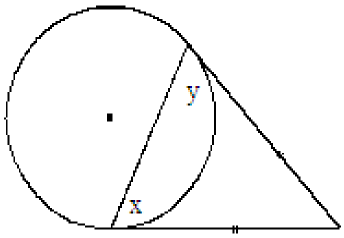
$x = 90°$

6) The angle between a tangent and a radius is a right angle.



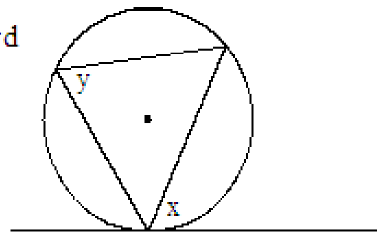
$x = 90°$

7) The two tangents from a point to a circle are equal in length.
Hence the angles between the tangents and the chord are equal.



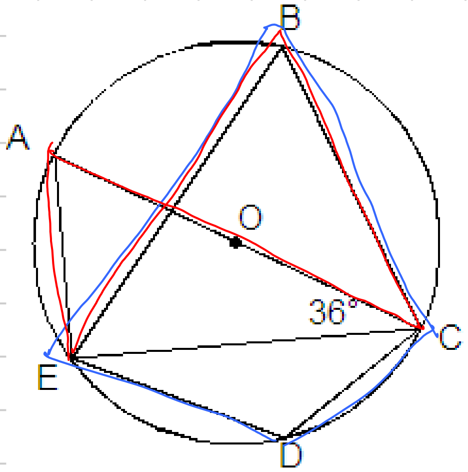
$x = y$

8) The angle between a tangent and a chord is equal to the angle in the alternate segment of the circle.



$x = y$

Examples



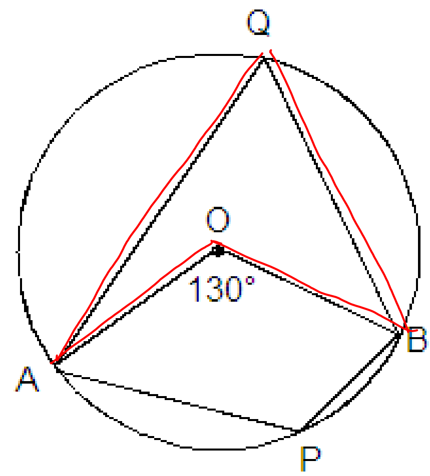
1) Find the angles:
(a) AEC (b) EAC
(c) EBC (d) EDC

(a) $A\hat{E}C = 90°$ (an angle in a semicircle)

(b) $E\hat{A}C = 180 - (90+36) = 180 - 126 = 54°$ (angles in a triangle add to 180°)

(c) $E\hat{B}C = 54°$ (angles in same segment)

(d) $E\hat{D}C = 180 - 54° = 126°$ (opposite angles of cyclic quadrilateral)



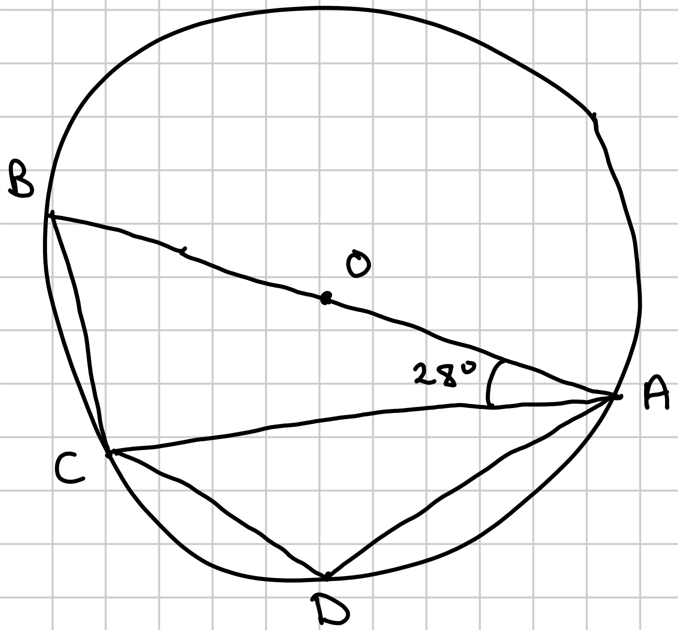
2) Find the angle APB

We need to work out AQB first :—

$A\hat{Q}B = 65°$ (angle at centre is twice angle at circumference)

$\therefore$ $A\hat{P}B = 180 - 65° = \underline{\underline{115°}}$ (opposite angles of a cyclic quadrilateral)

③ Find, giving reasons, angle $A\hat{D}C$.



$A\hat{C}B = 90°$ (angle in a semicircle)

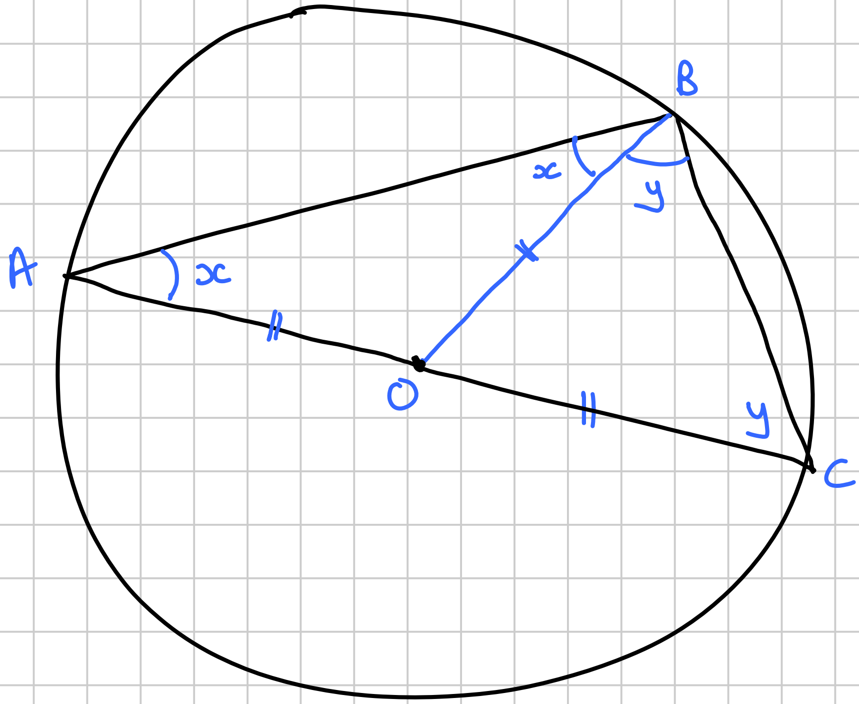
$A\hat{B}C = 62°$ (<s in a Δ add up to 180°)

$A\hat{D}C = 118°$ (opp <s of cyclic quad add to 180°)

PROOFS OF CIRCLE THEOREMS

Note Title 11/06/2012

Proof that an angle in a semicircle is a right angle



$O\hat{A}B = x$ (because Δ AOB is isosceles)

$O\hat{C}B = y$ (" Δ OBC " ")

$x + y + (x + y) = 180$ (angles in Δ ABC add to 180)

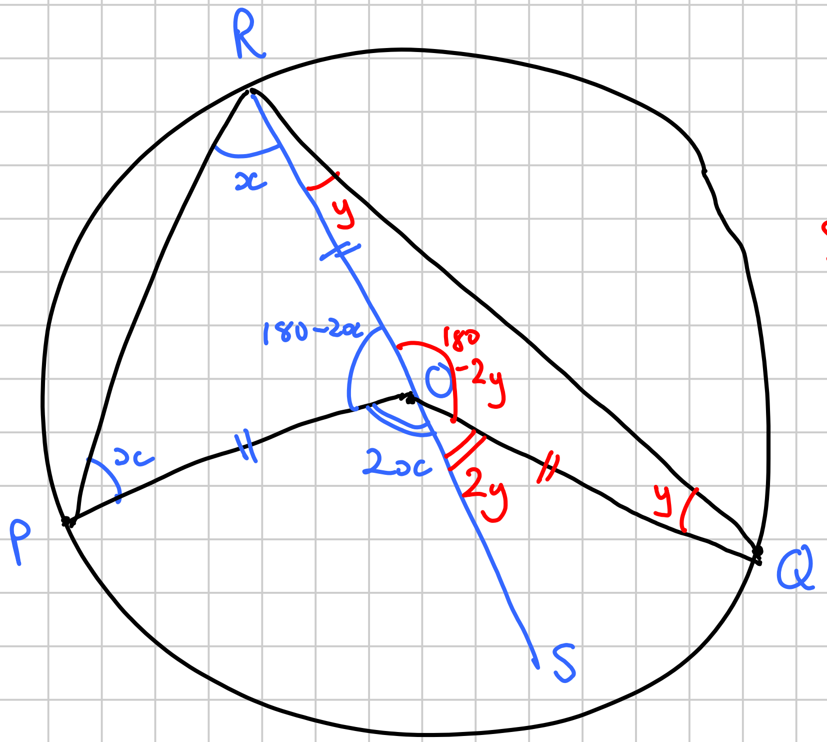
$2x + 2y = 180$

$2(x+y) = 180$

$(\div 2)$ $\quad x + y = 90^\circ$

$\underline{\underline{A\hat{B}C = 90^\circ}}$

Proof that angle at centre is twice angle at circumference



$$P\hat{O}S = 180 - (180 - 2x)$$
$$= 180 - 180 + 2x$$
$$= 2x$$

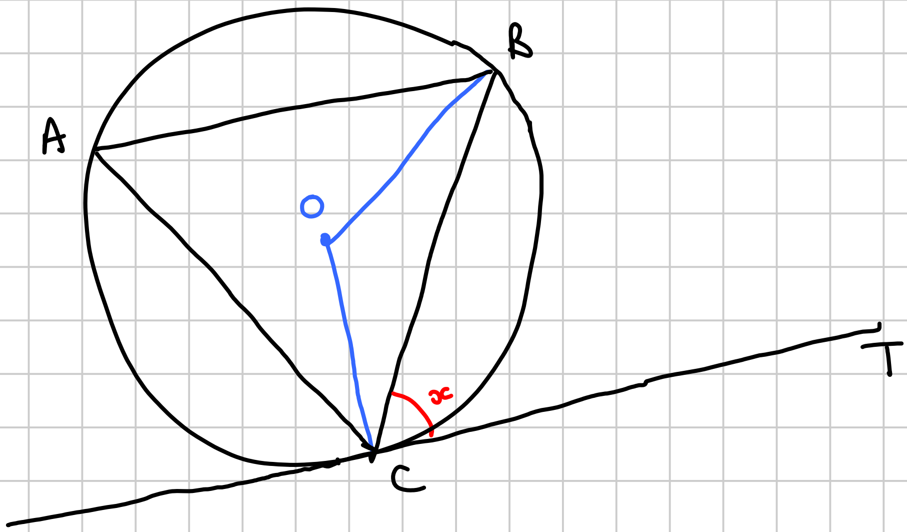
Similarly $P\hat{Q}S = 2y$

So $P\hat{R}Q = x + y$

$$P\hat{O}Q = 2x + 2y$$
$$= 2(x+y)$$
$$= 2 \times PRQ$$

Note Title 14/06/2012

② Prove the alternate segment theorem (no 8 on list)



ie, given that $B\hat{C}T = x°$, prove that $B\hat{A}C = x°$

$B\hat{C}O = 90 - x$ (angle between tangent and radius is 90°)

$C\hat{B}O = 90 - x$ ($\triangle$ OBC is isoscles)

$B\hat{O}C = 180 - 2(90 - x)$ (angles in $\triangle$ add to 180°)

$= 180 - 180 + 2x$

$= 2x$

$B\hat{A}C = x$ (angle at circumference is half angle at centre of circle)

Q.E.D. (Quod erat demonstrandum)