Note Title 23/06/2010

TRIGONOMETRY IN NON-RIGHT-ANGLED TRIANGLES

Sine and Cosine of Obtuse angles

The sine of an obtuse angle x is the same as the sine of the acute angle $180 - x$.

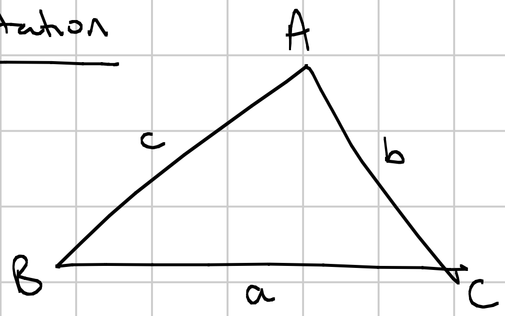
e.g $\sin 150 = \frac{1}{2}$ $\sin 30 = \frac{1}{2}$

$\sin 140 = 0.642\ldots$ $\sin 40 = 0.642\ldots.$

The cosine of an obtuse angle x is the NEGATIVE of the cosine of the acute angle $180 - x$

e.g $\cos 140 = -0.766$ $\cos 40 = 0.766$

Notation



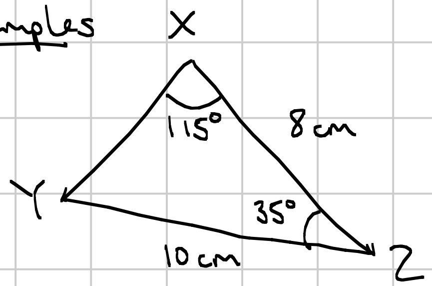
We use capital letters for angles

We use lower case a for the length of the side OPPOSITE angle A, etc

The Sine Rule (for finding a side)

In any triangle, $$\frac{a}{\sin A} = \frac{b}{\sin B} \quad \left(= \frac{c}{\sin C} \right)$$

Examples

1. 

Find XY

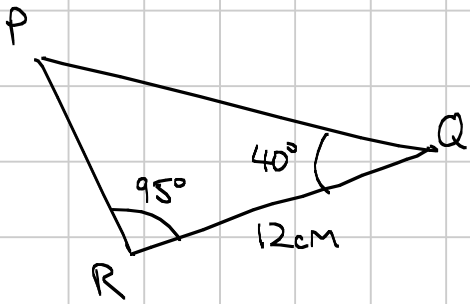
$$\frac{XY}{\sin 35^\circ} = \frac{10}{\sin 115^\circ}$$

$$(\times \sin 35) \qquad (\times \sin 35)$$

$$XY = \frac{10}{\sin 115^\circ} \times \sin 35^\circ$$

$$= \underline{\underline{6.33\text{cm}}}$$

(2)



$$\left(\hat{P} = 180 - 95 - 40 = 45^\circ\right)$$

Find PQ

$$\frac{PQ}{\sin 95} = \frac{12}{\sin 45^\circ}$$

$$(\times \sin 95) \qquad (\times \sin 95)$$

$$PQ = \frac{12}{\sin 45} \times \sin 95$$

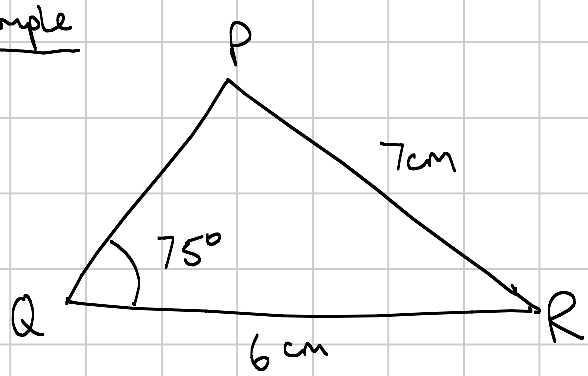
$$= \underline{\underline{16.9\text{cm}}}$$

The Sine Rule (for finding an angle)

To find an angle, we turn the sine rule upside down :-

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

Example



Find $\hat{P}$

$$\frac{\sin P}{6} = \frac{\sin 75^\circ}{7}$$

$$(\times 6) \qquad (\times 6)$$

$$\sin P = \frac{\sin 75}{7} \times 6$$

$$= 0.8279$$

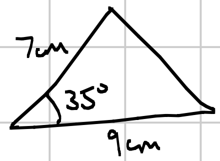
$$P = \sin^{-1}(0.8279)$$

$$= \underline{\underline{55.9^\circ}}$$

There are two situations in which the Sine Rule cannot be used :—

① If we know two sides and the angle between them

e.g



② If we know all 3 sides but no angles.

[In both of these we do NOT know a side and the angle opposite, so can't get started on the sine rule.]

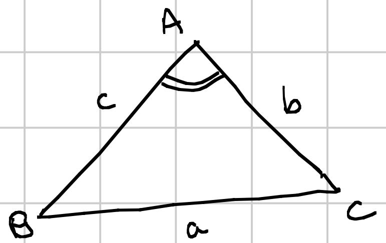
For these situations we need :

The Cosine Rule

This is an enhanced version of Pythagoras Theorem, for use in non-right-angled triangles.

Cosine Rule version 1

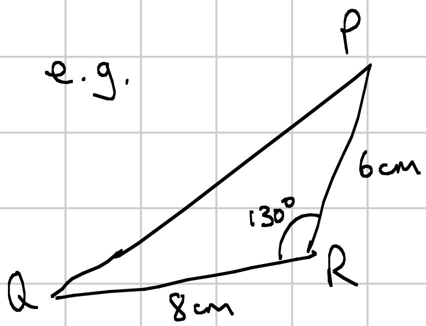
– for two sides and angle between



$$a^2 = b^2 + c^2 - 2bc\cos A$$

(it is the RELATIVE POSITION of the sides and angle which is important)

e.g.



Find PQ

$$PQ^2 = 8^2 + 6^2 - 2 \times 8 \times 6 \times \cos 130°$$

$$= 161.7$$

$$PQ = \sqrt{161.7}$$

$$= \underline{\underline{12.7\text{ cm}}}$$

Cosine Rule version 2 – for when we know all 3 sides and no angles

We need to make $\cos A$ the subject of the cosine rule :–

$$a^2 = b^2 + c^2 - 2bc\cos A$$

← IS on formula sheet

$(+2bc\cos A)$ $\quad$ $(+2bc\cos A)$

$$a^2 + 2bc\cos A = b^2 + c^2$$

$(-a^2)$ $\quad$ $(-a^2)$

$$2bc\cos A = b^2 + c^2 - a^2$$

$(\div 2bc)$ $\quad$ $(\div 2bc)$

$$\boxed{\cos A = \frac{b^2 + c^2 - a^2}{2bc}}$$

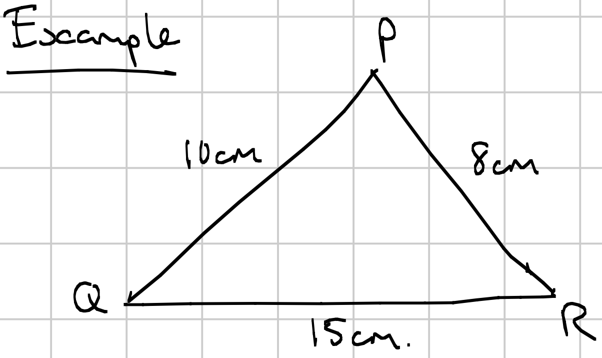
(It is relative positions which are important – a is opposite $\hat{A}$)

← Is NOT on formula sheet.

Either learn it

or learn how to get it from the other one.

Example



Find $\hat{P}$

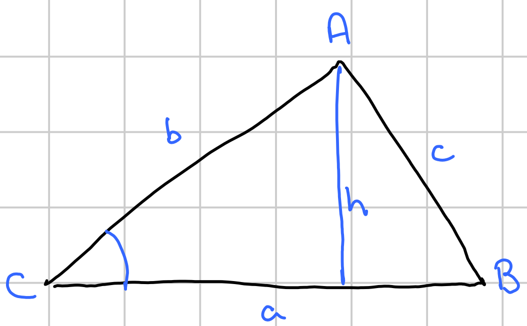
$$\cos\hat{P} = \frac{8^2 + 10^2 - 15^2}{2 \times 8 \times 10}$$

$$\cos\hat{P} = -0.38125$$

$$\hat{P} = \cos^{-1}(-0.38125)$$

$$= \underline{\underline{112.5^\circ}}$$

Area of a Triangle



Area of $\Delta ABC = \frac{1}{2}ah$

But $\frac{h}{b} = \sin C$, so $h = b\sin C$

$$\boxed{\text{So area of } \Delta ABC = \tfrac{1}{2}ab\sin C}$$

Example: Area of triangle above

$= \frac{1}{2} \times 10 \times 8 \times \sin 112.5^\circ = 37.0\text{cm}^2$