

SIMILAR SHAPES - AREA AND VOLUME

Note Title

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- The ratio of the AREAS of similar shapes is the SQUARE of the ratio of their LENGTHS
- The ratio of the VOLUMES of similar shapes is the CUBE of the ratio of their LENGTHS

Examples

① A photograph has a height of 6cm, and an area of 27cm^2 . Find the area of an enlargement of the photo with a height of 9cm.

$$\begin{aligned}\text{Ratio of lengths} &= 6:9 \\ &= 2:3\end{aligned}$$

$$\begin{aligned}\text{Ratio of areas} &= 2^2:3^2 \\ &= 4:9\end{aligned}$$

$$\text{Area of enlargement} = 27 \times \frac{9}{4} = 60.75\text{cm}^2$$

larger number on top
because we want to
increase the area

② Model houses come in small and large sizes. The height of a small model is 8cm, and of a large model is 10cm. If the volume of a large model is 1500cm^3 , what is the volume of a small model?

$$\begin{aligned}\text{Ratio of lengths} &= 8:10 \\ &= 4:5\end{aligned}$$

$$\begin{aligned}\text{Ratio of volumes} &= 4^3:5^3 \\ &= 64:125\end{aligned}$$

$$\text{Volume of small model} = 1500 \times \frac{64}{125} = \underline{\underline{768\text{cm}^3}}$$

Sometimes we need to work backwards :-

- the ratio of lengths is the SQUARE ROOT of the ratio of the areas
- the ratio of lengths is the CUBE ROOT of the ratio of the volumes

(3) A paddling pool has an area of 32 m^2 and a width of 5 m . What is the width of a similar paddling pool with an area of 18 m^2 ?

$$\begin{aligned} \text{Ratio of areas} &= 18 : 32 \\ &= 9 : 16 \end{aligned}$$

$$\begin{aligned} \text{Ratio of lengths} &= \sqrt{9} : \sqrt{16} \\ &= 3 : 4 \end{aligned}$$

$$\begin{aligned} \text{Width of smaller pool} &= 5 \times \frac{3}{4} \\ &= \underline{\underline{3.75 \text{ m}}} \end{aligned}$$

(4) Two balls have volumes of 10 cm^3 and 80 cm^3 . The surface area of the larger one is 32 cm^2 . What is the surface area of the smaller one?

NOTE: We cannot convert the ratio directly from volume to area - we need to go via ratio of lengths

$$\begin{aligned}\text{Ratio of volumes} &= 10 : 80 \\ &= 1 : 8\end{aligned}$$

$$\begin{aligned}\text{Ratio of lengths} &= \sqrt[3]{1} : \sqrt[3]{8} \\ &= 1 : 2\end{aligned}$$

$$\begin{aligned}\text{Ratio of areas} &= 1^2 : 2^2 \\ &= 1 : 4\end{aligned}$$

$$\begin{aligned}\text{Surface area of small ball} &= 32 \times \frac{1}{4} \\ &= \underline{\underline{8 \text{ cm}^2}}\end{aligned}$$