**Solid Shapes - Volume and Surface Area**

**Prism**

A prism is a solid shape with a **constant cross-section** (i.e., if we cut through it anywhere the cut looks the same).

**Example**

![Diagram of a prism]

Volume = area of cross-section \( \times \) length

\[
\begin{align*}
\text{Area of cross-section} &= \text{Area } A + \text{Area } B \\
&= (4 \times 2) + (5 \times 2) \\
&= 18 \text{ cm}^2 \\
\text{Volume} &= 18 \times 30 \\
&= 540 \text{ cm}^3
\end{align*}
\]

**Cylinder**

A cylinder is a prism with a circular cross-section.

So area of cylinder = \( \pi r^2 \times \text{length or height} \)

\[
\pi r^2 h
\]

Also, curved surface area = \( 2\pi rh \)

**Example** Find the volume and surface area of a tin with diameter 10 cm and height 12 cm

Volume = \( \pi \times 5^2 \times 12 = 942 \text{ cm}^3 \)

Curved surface area = \( 2 \times \pi \times 5 \times 12 = 377 \text{ cm}^2 \)

Area of top = \( \pi \times 5^2 = 78.5 \text{ cm}^2 \)

Total surface area = \( 377 + (2 \times 78.5) = 534 \text{ cm}^2 \)
Pyramids  A pyramid can have many different shapes of base.

Volume of pyramid = \( \frac{1}{3} \times \text{area of base} \times \text{perpendicular height} \)

Cone  A cone is a pyramid with a circular base

Volume = \( \frac{1}{3} \pi r^2 h \)

Curved surface area = \( \pi rl \)

(\text{where } l = \text{the slant height of the cone})

Also, by Pythagoras, \( h^2 + r^2 = l^2 \)

Example

(a) Find the total surface area (including the base)

Curved S.A = \( \pi \times 10 \times 26 = 816.81 \text{cm}^2 \)

Base = \( \pi \times 10^2 = 314.16 \text{cm}^2 \)

Total SA = \( 1230.97 \text{cm}^2 \)

(b) Find the volume

First find \( h \):

\( h^2 + 10^2 = 26^2 \)

\( (-10^2) \quad (-10^2) \)

\( h^2 = 26^2 - 10^2 \)
\[ \frac{h^2}{h^2} = 676 - 100 \\
\frac{h^2}{h} = 576 \\
h = \sqrt{576} = 24 \text{ cm} \]

\[ \text{Volume} = \frac{1}{3} \pi \times 10^2 \times 24 \]
\[ = \frac{2513}{3} \text{ cm}^3 \]

** Sphere **

\[ \text{Volume} = \frac{4}{3} \pi r^3 \]

\[ \text{Surface Area} = 4\pi r^2 \]

** Example **

The volume of a sphere is 1000 cm\(^3\). What is its surface area?

First find \( r \)

\[ \frac{4}{3} \pi r^3 = 1000 \]

\[(\times 3) \quad (\times 3)\]

\[4\pi r^3 = 3000 \]

\[(\div 4) \quad (\div 4)\]

\[\pi r^3 = 750 \]

\[(\div \pi) \quad (\div \pi)\]

\[r^3 = 238.73 \ldots \]

\[(\text{cube root}) \quad (\text{cube root})\]

\[r = 6.2034 \]

\[ \text{Surface area} = 4\pi \times 6.2034^2 \]
\[ = 483.6 \text{ cm}^2 \]