

## LOCI AND CONSTRUCTIONS

1) A field ABCD is in the shape of a rectangle 80m by 50m. Some treasure is buried at a point equidistant from corners A and C, and 35m from corner D. Make a scale drawing on a scale of 1:1000, and use ruler and compasses to find the location of the treasure. By measuring, write down the distance of the treasure from corner B.

2) Repeat Q1; this time the treasure is equidistant from lines AB and BC, and 60m from A.

3) Draw an acute angled triangle ABC, leaving some room around it.

(a) Construct the locus of points equidistant from A and B, and the locus of points equidistant from B and C.

(b) What can you say about the point where these lines cross?

(c) Confirm your answer to (b) by drawing the locus of points equidistant from A and C.

(d) The point where all three lines meet is called the CIRCUMCENTRE of the triangle. Now place the point of your compass on the circumcentre. By adjusting the radius of your compass, you should find that you can draw a circle which passes through all three corners of the triangle. This is called the CIRCUMCIRCLE.

4) Repeat question 3, but this time make one of the angles of your original triangle obtuse. What difference does this make?

5) Can you think of a way of making the circumcentre of your triangle fall exactly ON one of the sides of the triangle?

6) Draw a fairly large triangle ABC. Bisect each of the corner angles. You should find that all three bisectors meet at one point, called the INCENTRE of the triangle.

Now place the point of your compass on the incentre. By adjusting the radius of your compass, you should find that you can draw a circle which just touches all three sides of the triangle. This is called the INCIRCLE.

7) Construct angles of the following sizes. (Check them using a protractor.)

(a)  $30^\circ$  (b)  $90^\circ$  (c)  $15^\circ$  (d)  $45^\circ$  (e)  $75^\circ$  (f)  $120^\circ$

8) Construct a triangle ABC in which  $AB = 10\text{cm}$ , angle  $BAC = 60^\circ$  and angle  $ABC = 45^\circ$ . Measure and write in the lengths AC and BC.

9) Construct a triangle XYZ in which  $XY = 6\text{cm}$ , angle  $YXZ = 15^\circ$  and angle  $XYZ = 105^\circ$ . Measure and write in the lengths XZ and YZ.

10) Draw a line AC 8cm long. Construct the perpendicular bisector of this line, and mark the midpoint of AC as M. Mark points B and D on the perpendicular bisector so that  $BM = DM = 5\text{cm}$ , and draw the rhombus ABCD. What is the area of this rhombus?

11) Draw PQ 12cm long. Construct angle PQR to be  $30^\circ$ , and mark R so that  $QR = 8\text{cm}$ . Drop a perpendicular from R to the line PQ, and mark the intersection with PQ as S. Join PR, and measure and mark on the lengths PR, RS, and PS, and all the angles in the diagram.

## RULER AND COMPASS CONSTRUCTIONS II

1) The construction for a perpendicular bisector can be used simply to find the midpoint of a line. Draw a triangle ABC and find the midpoint of each side, marking them P, Q, and R. Join each corner to the opposite midpoint. (Each of these lines is called a MEDIAN of the triangle.) They should all meet in a point, called the CENTROID of the triangle.

2) The CENTROID of a triangle is its centre of gravity. Try drawing a triangle on card, and finding the centroid. Then cut the triangle out, and balance it on the point of your compass. In theory you should find that it balances exactly on the centroid - but it very hard to make this work!

3) Draw an acute-angled triangle ABC and drop a perpendicular from each corner to the opposite side. (All three lines, which are called ALTITUDES, should meet at a point called the ORTHOCENTRE.) Measure and mark on the length of each side, and the length of each perpendicular. Now by turning the triangle round, you can work out its area as "half base times height" in 3 different ways. Do this, and compare your answers.

4) The "NINE POINT CIRCLE". This is a famous construction discovered by the ancient Greeks.

Draw a fairly large circle ABC. Now mark in the three medians AP, BQ and CR, as in Q1. Label the centroid of the triangle M. Also mark in the three altitudes AX, BY and CZ, as in Q3. Label the orthocentre O. Now find the midpoints of the lines OA, OB and OC, labelling them J, K and L.

Now (if you can still find them amid the mass of construction lines!), mark with a coloured dot the points P, Q, R, X, Y, Z, J, K, L. You should see that they all lie on a circle! To find the centre, draw in the lines JP, KQ and LR (which should all meet at a point). Finally, draw in the NINE POINT CIRCLE.