PROPORTIONALITY

Note Title 20/04/2010

The sign $\propto$ means "is proportional to"

A statement such as "$y \propto x$" can always be replaced by "$y = kx$"

The constant k is called a "constant of proportionality"

Example An object is dropped from an aircraft. The distance (s) which it has fallen is proportional to the square of the time (t) for which it has been falling. After 4 seconds it has fallen 80 m.

(a) Find a formula for s in terms of t

Write in symbols: $s \propto t^2$

Convert to a formula: $s = kt^2$

To find k, substitute in the given values

$80 = k \times 16$

$k = 5$

Then put this value of k into the formula

$s = 5t^2$

(b) How far will the object fall in 8 seconds?

Substitute $t = 8$: $s = 5 \times 8^2 = 320\text{m}$

(c) How long would it take for the object to fall 500m?

Substitute $s = 500$: $s = 5t^2$

$500 = 5t^2$

$(\div 5) \quad (\div 5)$

$100 = t^2$

$(\sqrt{\ }) \quad (\sqrt{\ })$

$10 = t$

It would take 10 seconds

Inverse Proportionality

If y is inversely proportional to x, then doubling x causes y to be halved.

We write this as $y \propto \frac{1}{x}$

which turns in to $y = \frac{k}{x}$

Example The time (T) to do a job is inversely proportional to the number of people (n) working on it. With 3 people the job will take 6 hours. How long will it take with 10 people?

Write in symbols $T \propto \frac{1}{n}$

Write with "= and k" $T = \frac{k}{n}$

Use the numbers given to find k:

$$6 = \frac{k}{3}$$

(x3) (x3)

$$18 = k$$

Write the formula replacing k by the number

$$T = \frac{18}{n}$$

Now we can use this formula to answer questions

If $n = 10$, $T = \frac{18}{10} = 1.8$ hours

(b) How many people would be needed to do the jobs in 1 hour?

If $T = 1$,

$$1 = \frac{18}{n}$$

$(\times n)$ $(\times n)$

$$n = 18 \text{ people}$$