

PERCENTAGES

Note Title

20/10/2010

To convert a fraction or a decimal to a percentage, multiply by 100.

To convert a percentage to a fraction or a decimal, divide by 100.

Examples

① Convert each of the following to a percentage

(a) 0.36 $0.36 \times 100 = 36\%$

(b) 0.074 $0.074 \times 100 = 7.4\%$

(c) $\frac{9}{20}$ $\frac{9}{20} \times 100 = 45\%$

(d) $\frac{11}{40}$ $\frac{11}{40} \times 100 = \frac{55}{2} = 27\frac{1}{2}\%$

② Convert to a decimal

(a) 12.5% $12.5 \div 100 = 0.125$

(b) 130% $130 \div 100 = 1.3$

③ Convert to a fraction

(a) 48% $\frac{48}{100} = \frac{12}{25}$

(b) $13\frac{1}{3}\%$ $\frac{13\frac{1}{3}}{100} \xrightarrow{\times 3} \frac{40}{300} = \frac{4}{30} = \frac{2}{15}$

④ Expressing one number as a percentage of another

e.g. Out of 640 students in a school, 96 come to school by bike.
What percentage of the students come by bike?

$$\frac{96}{640} \times \frac{100}{1} = 15\%$$

⑤ Finding a percentage of a number

- In this situation, "OF" means \times
- We can change the percentage to a fraction or a decimal, whichever we prefer.

e.g. ① Out of 650 students in a school, 24% come to school by car.
How many students come by car?

ie, find 24% of 650

$$\frac{24}{100} \times 650 = \underline{\underline{156}} \text{ students}$$

$$\text{[OR } 0.24 \times 650 = 156 \text{]}$$

② The price of a computer has increased by 12%.
If it was £425, what is it now?

$$\begin{aligned} 12\% \text{ of } 425 &= 0.12 \times 425 \\ &= \underline{\underline{£51}} \end{aligned}$$

$$\text{So new price} = 425 + 51 = \underline{\underline{£476}}$$

[But we will see a better way to do this next lesson.]

Percentage Increase and Decrease

To increase or decrease an amount by a certain percentage, we multiply by a **MULTIPLYING FACTOR**.

Examples

① The price of a computer has increased by 12%.
If it was £425 before, what is it now?

Original price = 100% so new price = 112%

Find 112% of £425

$$1.12 \times 425 = \underline{\underline{£476}}$$



[To increase by 12%, the multiplying factor is 1.12]

② The population of a town has decreased by 28%.
If it was 115000, what is it now?

Original pop = 100% so New pop = 72%

Find 72% of 115000

$$0.72 \times 115000 = \underline{\underline{82800}} \text{ people}$$

③ Write down the multiplying factor to:

- | | |
|-----------------------------------|-------|
| (a) Increase by 8% | 1.08 |
| (b) Decrease by $5\frac{1}{2}\%$ | 0.945 |
| (c) Increase by $17\frac{1}{2}\%$ | 1.175 |

Finding what percentage A is of B

To do this, we write

$$\frac{A}{B} \times \frac{100}{1}$$

Examples

- ① Out of 540 pupils in a school, 144 come to school by car. What percentage come by car?

$$\frac{144}{540} \times \frac{100}{1} = \frac{80}{3} = 26\frac{2}{3} \\ = \underline{\underline{26\frac{2}{3}\%}}$$

- ② Alice receives £7.20 pocket money per week, of which she spends £2.70 on sweets. What percentage does she spend on sweets?

$$\frac{270}{720} \times \frac{100}{1} = \frac{75}{2} \\ = \underline{\underline{37.5\%}}$$

Profit and loss

To work out a percentage profit or loss we calculate

$$\% \text{ profit} = \frac{\text{actual profit}}{\text{ORIGINAL (cost) price}} \times \frac{100}{1}$$

Examples

- ① I bought a Beatles LP for £5 in 1969, and sold it for £20 last year. What percentage profit did I make?

$$\text{Actual profit} = £20 - £5 = £15$$

$$\text{Percentage profit} = \frac{15}{5} \times \frac{100}{1} \\ = \underline{\underline{300\%}}$$

- ② I bought a car for £8000. 3 years later I sold it for £1800. What was my percentage loss?

$$\text{Actual loss} = 8000 - 1800 = \text{£}6200$$

$$\begin{aligned}\text{Percentage loss} &= \frac{6200}{8000} \times \frac{100}{1} \\ &= \underline{\underline{77.5\%}}\end{aligned}$$

Finding the original amount

Examples

- ① A shop sells a camera for £132, making a profit of 10%. How much does the shop pay for the camera?

$$\text{Original price} = x \quad \text{Multiplying factor} = 1.1$$

$$1.1x = 132$$

$$x = \frac{132}{1.1} = \underline{\underline{\text{£}120}}$$

- ② The population of a village fell by 15% between 2000 and 2010. In 2010 it was 5270. What was it 2000?

$$\text{Population in 2000} = x \quad \text{Multiplying factor} = 0.85$$

$$0.85x = 5270$$

$$x = \frac{5270}{0.85} = \underline{\underline{6200}}$$

Compound Percentages

Examples

- ① I invest £10000 in a fixed-rate bond for 6 years at 5% per year interest.
How much will I receive back after 6 years?

Multiplying factor to increase by 5% is 1.05

$$\text{Answer is } \pounds 10000 \times 1.05^6 = \pounds 13400.96$$

- ② A car depreciates (goes down in value) by 30% in its first year, and 15% in each of the next 4 years.
I buy a new car for £12000. How much is it worth after 5 years?

$$12000 \times 0.7 \times 0.85^4 = \pounds 4385$$

- ③ The population of a town increases by 30%, then decreases by 25%. What percentage has the population increased by overall?

$$\begin{aligned} & P \times 1.3 \times 0.75 \\ & = P \times 0.975 \end{aligned}$$

(Trick question!) Population has decreased by 2.5%.