**VECTORS**

A vector has both:
- Magnitude (size, length)
- Direction

A vector can be represented as:

\[ \overrightarrow{AB} \] (geometrical)

or

\[
\begin{pmatrix}
3 \\
2
\end{pmatrix}
\] (numerical - provided we have coordinates to use)

or

\[ v \] (algebraic - underlined to show it is a vector - in textbooks, bold type is used instead of underlining)

---

**Equal Vectors**

If two vectors have the same length and direction they are equal even if they start at different places.

\[ v \neq w \] because although they are the same length, they are in different directions

\[ v = d \]
The **Negative of a Vector** has the same length but in the opposite direction.

\[ \vec{V} \quad \rightarrow \quad -\vec{V} \]

A **multiple of a Vector** has the same direction but a different length.

\[ 2\vec{V} \]

**Adding Vectors** is done by placing them "nose-to-tail" and drawing a single vector from the start to the finish.

\[ \vec{V} \quad \rightarrow \quad \vec{V} + \vec{V} \]

**Numerically:**

\[
\left( \frac{3}{2} \right) + (-4) = (4, -2)
\]
ABCE is a parallelogram. \( \vec{AE} = \vec{v} \), \( \vec{AB} = \vec{w} \).
Express the following vectors in terms of \( \vec{v} \) and/or \( \vec{w} \).

(a) \( \vec{BC} = \vec{v} \)

(b) \( \vec{BA} = -\vec{w} \)

(c) \( \vec{AB} = 2\vec{v} \)

(d) \( \vec{AC} = \vec{AB} + \vec{BC} \)
    \[= \vec{w} + \vec{v} \]

(e) \( \vec{BE} = \vec{BC} + \vec{CE} \)
    \[= \vec{v} + (-\vec{w}) \]
    \[= \vec{v} - \vec{w} \]

(f) \( \vec{CD} = \vec{CE} + \vec{ED} \)
    \[= -\vec{w} + \vec{v} \] \(\text{or} \quad \vec{v} - \vec{w} \)
In this diagram, A divides OP in the ratio 2:1.
B divides OQ in the ratio 1:3.
Express the following vectors in terms of \( a \) and \( b \):

(a) \( \overrightarrow{AP} = \frac{1}{2}a \)
(b) \( \overrightarrow{AB} = \overrightarrow{A} - \overrightarrow{B} \)
(c) \( \overrightarrow{OQ} = 4\overrightarrow{b} \)
(d) \( \overrightarrow{PO} = -\frac{1}{2}a \)
(e) \( \overrightarrow{PQ} = \overrightarrow{P} - \overrightarrow{Q} \)
(f) \( \overrightarrow{PN} = \frac{2}{3}\overrightarrow{PQ} \)

Using Vectors to prove Geometric Facts:

ABCD is a parallelogram.
M is the midpoint of AB.
NC = 2AN.
AD = \( \overrightarrow{V} \) and AM = \( \overrightarrow{W} \).

Express in terms of \( \overrightarrow{V} \) and/or \( \overrightarrow{W} \):

(a) \( \overrightarrow{AC} = \overrightarrow{AD} + \overrightarrow{DC} \)
(b) \( \overrightarrow{AN} = \frac{1}{3} \overrightarrow{AC} \)
(c) \[ \overrightarrow{MN} = \overrightarrow{MA} + \overrightarrow{AN} \]
\[ = -\overrightarrow{w} + \frac{1}{3} \overrightarrow{v} + \frac{2}{3} \overrightarrow{w} \]
\[ = \frac{1}{3} \overrightarrow{v} - \frac{1}{3} \overrightarrow{w} \]
\[ = -\frac{1}{3} \overrightarrow{w} + \frac{1}{3} \overrightarrow{v} \] (is the same)

(d) \[ \overrightarrow{ND} = \overrightarrow{NA} + \overrightarrow{AB} \]
\[ = -\frac{1}{2} \overrightarrow{v} - \frac{2}{3} \overrightarrow{w} + \overrightarrow{v} \]
\[ = \frac{2}{3} \overrightarrow{v} - \frac{2}{3} \overrightarrow{w} \]

(e) Using your answers to (c) and (d), state two facts about \( M, N \) and \( D \).

\[ \overrightarrow{ND} = 2 \overrightarrow{MN} \]

We deduce that:

- the distance \( ND = 2 \times \) the distance \( MN \)
- \( ND \) is in the same direction as \( MN \)

\[ \text{ie } M, N \text{ and } D \text{ lie in a straight line} \]

**If** \( \overrightarrow{AB} = k \times \overrightarrow{CB} \), we can deduce two things

- one about the **LENGTHS**
- one about the **DIRECTIONS**

- either that points are in a line, or that lines are parallel (\( k \) could be any number)
Magnitude of a Vector

If a vector is given in the form \((a, b)\), then we can find the magnitude (i.e., length) of the vector by using Pythagoras:

\[
\text{magnitude of } \mathbf{v} = \sqrt{a^2 + b^2}
\]

The magnitude can be written as \(|\mathbf{v}|\).

Example: If \(\mathbf{v} = (-3, 2)\), find \(|\mathbf{v}|\).

\[
|\mathbf{v}| = \sqrt{(-3)^2 + 2^2} = \sqrt{9 + 4} = \sqrt{13}
\]

\((\approx 3.6 \text{ units to 1 dp})\).