Quadratic Expressions and Equations
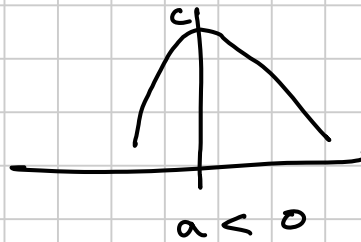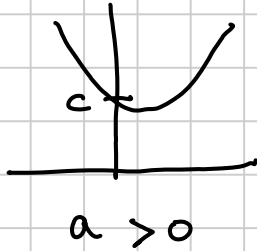
A quadratic expression is of the form

$$a x^2 + bx + c \qquad (a \neq 0)$$

Its graph is a parabola

$a > 0$ $a < 0$

It can be useful to write the expression in other ways :—

① **Factorise**

e.g. $3x^2 - 13x - 10$

$\left(\begin{array}{l} 3 \times -10 = -30 \qquad \text{factors of } -30 \text{ which add to } -13 \\ \qquad\qquad\qquad\qquad \text{are} \qquad -15 \text{ and } 2. \end{array} \right)$

$$= 3x^2 - 15x + 2x - 10$$

Factorise in pairs

$$= 3x(x - 5) + 2(x - 5)$$

$$= \underline{\underline{(3x + 2)(x - 5)}}$$
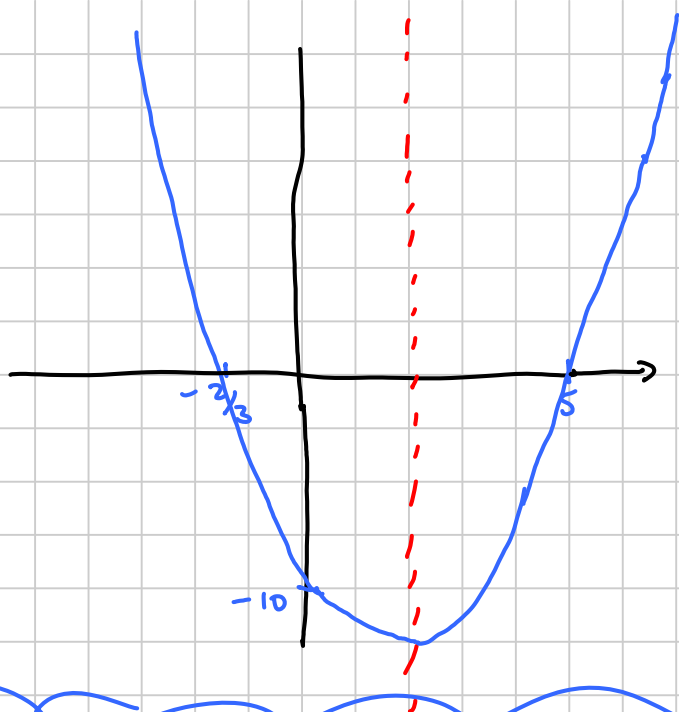
From this we can solve $3x^2 - 13x - 10 = 0$
by saying

$$3x + 2 = 0 \qquad\qquad \text{or} \qquad x - 5 = 0$$
$$x = \frac{-2}{3} \qquad\qquad \text{or} \qquad \underline{\underline{x = 5}}$$

And this shows where the graph of $y = 3x^2 - 13x - 10$ crosses the x-axis :

(graph with labels $-\frac{2}{3}$, 5 on x-axis, -10 on y-axis, blue parabola)

② <u>Completing the square</u>

Some quadratics factorise as a perfect square

e.g. $x^2 + 10x + 25 = (x + 5)^2$

If the quadratic is not a perfect square we can make an adjustment

e.g $x^2 + 6x + 4$

$$\left(\begin{array}{l} \text{Halve coefficient of } x: \quad \tfrac{1}{2} \text{ of } 6 = 3 \\ \text{Now square this :} \quad 3^2 = 9 \end{array} \right)$$

$= x^2 + 6x + 9 - 9 + 4$

$= (x + 3)^2 - 5$

Now we can use this to :
- Find the minimum value of $x^2 + 6x + 4$

   The square of a number is always non-negative
   So $(x + 3)^2 \geq 0$

$$(x + 3)^2 - 5 \geqslant -5$$

Min value of $x^2 + 6x + 4$ is -5 and occurs when $x = -3$.

- Solve
$$x^2 + 6x + 4 = 0$$
$$(x + 3)^2 - 5 = 0$$
$$(x + 3)^2 = 5$$
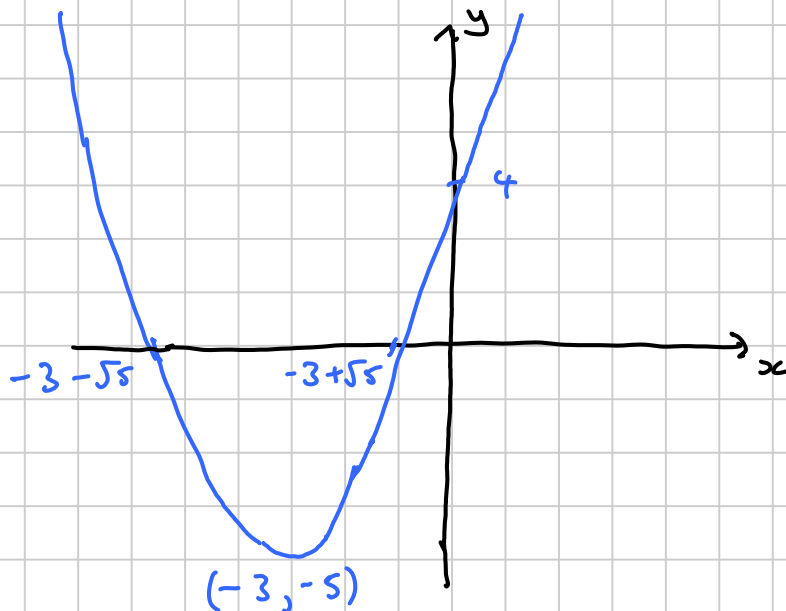
(Square root both sides, remembering the $\pm$ sign)
$$x + 3 = \pm \sqrt{5}$$
$$x = -3 + \sqrt{5} \text{ or } -3 - \sqrt{5}$$

(leave answers like this unless told otherwise)

- Sketch the graph of $y = x^2 + 6x + 4$

$-3 - \sqrt{5}$ $-3 + \sqrt{5}$ 4 $(-3, -5)$

More examples

① Write $x^2 - 8x + 20$ in the form $(x + q)^2 + r$
Hence :
    (a) Find the min value of $x^2 - 8x + 20$
    (b) Explain why $x^2 - 8x + 20 = 0$ has no solutions
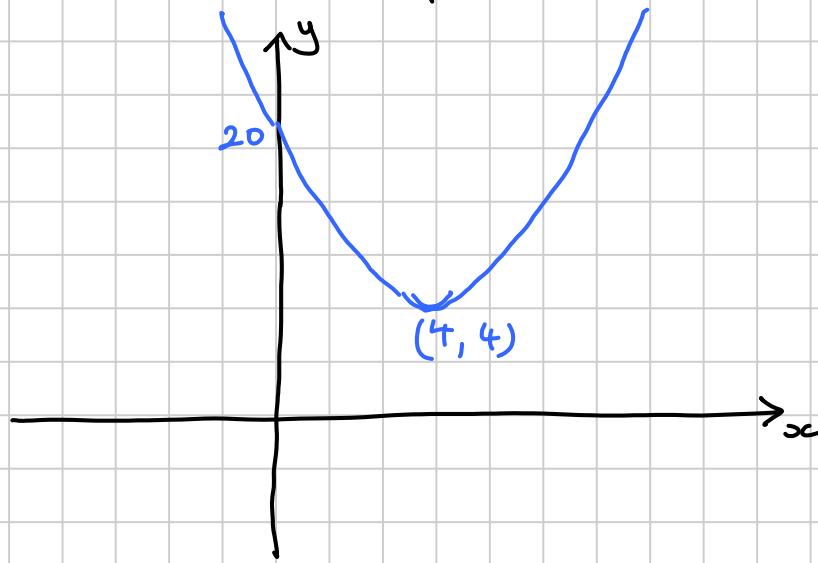   (c) Sketch the graph of $y = x^2 - 8x + 20$.

($\frac{1}{2}$ of $-8 = -4$, $(-4)^2 = 16$)
$$x^2 - 8x + 16 - 16 + 20$$
$$= (x - 4)^2 + 4$$

(a) Min value is 4, and occurs when $x = 4$

(b) Since $x^2 - 8x + 20$ cannot be less than 4, it can't be equal to 0.

(c)

② Write $3x^2 - 13x - 10$ in the form $p(x+q)^2 + r$

$$3\left(x^2 - \frac{13}{3}x - \frac{10}{3}\right)$$

$\left(\text{Half of } -\frac{13}{3} \text{ is } -\frac{13}{6}, \text{ and } \left(-\frac{13}{6}\right)^2 = \frac{169}{36}\right)$

$$= 3\left(x^2 - \frac{13}{3}x + \frac{169}{36} - \frac{169}{36} - \frac{10}{3}\right)$$

$$= 3\left(\left(x - \frac{13}{6}\right)^2 - \frac{169}{36} - \frac{120}{36}\right)$$

$$= 3\left(x - \frac{13}{6}\right)^2 - \frac{289}{12}$$

by Monday.

p 40 Ex 3.2 Q 2, 3bcd, 4, 5, 6