Note Title 24/11/2009

Solving Quadratic Equations

Quadratic equations are equations containing "x^2".
They normally have 2 solutions.

e.g.

$$x^2 = 5x - 6$$

Try $x = 2$, $x^2 = 4$ and $5x - 6 = 4$ ✓

Try $x = 3$, $x^2 = 9$ and $5x - 6 = 9$ ✓

So $x = 2$ and $x = 3$ are both solutions.

To solve quadratic equations, we first need to rearrange to make the Right Hand Side zero.

We will learn 2 methods :-

Method 1 : Factorizing

Examples

(1) $$x^2 = 5x - 6$$

$(+6)$ $(+6)$

$$x^2 + 6 = 5x$$

$(-5x)$ $(-5x)$

$$x^2 - 5x + 6 = 0$$ ← Right Hand Side is 0!

Factorize the left Hand Side

$$(x - 3)(x - 2) = 0$$

Now we know one of these brackets must equal 0 (because if two numbers multiply to zero, one of them has to be zero)

Either $x - 3 = 0$ or $x - 2 = 0$

$(+3)\ (+3)$ $(+2)\ (+2)$

$x = 3$ $x = 2$

(2) $x^2 + 2x = 35$

$(-35)\ (-35)$

$x^2 + 2x - 35 = 0$

$(x + 7)(x - 5) = 0$

Either $x + 7 = 0$ or $x - 5 = 0$

$(-7)\ (-7)$ $(+5)\ (+5)$

$x = -7$ $x = 5$

Method 2: Using the formula

The solutions of $ax^2 + bx + c = 0$ can be found using the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

One solution is found using the '+' part of the "±"
The other solution is found by using the '−' part.
It is probably safest to work out $\sqrt{b^2 - 4ac}$ on its own first.

Examples

① Solve $2x^2 - 5x - 8 = 0$ giving answers to 2 d.p.

$a = 2 \quad b = -5 \quad c = -8$

$\sqrt{(-5)^2 - 4 \times 2 \times (-8)} = 9.43398\ldots$

don't round off until the final answers.

remember to put brackets round negative numbers

$$\text{So} \quad x = \frac{-(-5) + 9.43398}{2 \times 2} = \underline{3.61} \ (2\text{dp})$$

($-b$: $-(-5)$; $+\sqrt{b^2-4ac}$: 9.43398; $2a$: 2×2)

$$\text{or} \quad x = \frac{-(-5) - 9.43398}{2 \times 2} = \underline{-1.11} \ (2\text{dp})$$

② $x^2 - 5x + 6 = 0$

$a = 1 \quad b = -5 \quad c = 6$

$\sqrt{b^2-4ac} = \sqrt{(-5)^2 - 4 \times 1 \times 6} = 1$

$$\text{So} \quad x = \frac{-(-5) + 1}{2 \times 1} = 3$$

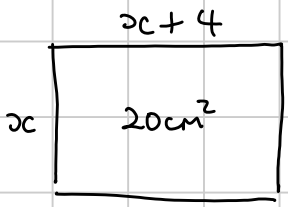
$$\text{or} \quad x = \frac{-(-5) - 1}{2 \times 1} = 2$$

which are the same answers as we found using factorisation!

- If the question says "give answers to 2 dp" or similar use method 2 (the formula) because factorization won't work.
- If the question says "factorize", use method 1 (factorization)
- Otherwise use whichever method seems easier.

Solving Problems using Quadratic Equations

Example 1 The length of a rectangle is 4 cm more than the width. The area is $20\,cm^2$. Find the width to 2 dp.



$$x(x+4) = 20$$
$$x^2 + 4x = 20$$
$$(-20) \qquad (-20)$$
$$x^2 + 4x - 20 = 0$$

$$a = 1 \quad b = 4 \quad c = -20$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4 \times 1 \times -20}}{2}$$

$$x = 2.90 \quad \text{or} \quad -6.90$$

x can't be negative since it is the width

The width is $\underline{\underline{2.90\ cm}}$

(2) In a right angled triangle the hypotenuse is 17cm. One of the other sides is 7cm longer than the other. Find the length of the shortest side.

By Pythagoras' theorem.

$$x^2 + (x+7)^2 = 17^2$$

$$x^2 + x^2 + 14x + 49 = 289$$

(-289) (-289)

$$2x^2 + 14x - 240 = 0$$

$(\div 2)$ $(\div 2)$

$$x^2 + 7x - 120 = 0$$

$(x+7)(x+7)$

$= x^2 + 7x + 7x + 49$

$= x^2 + 14x + 49$

x to -120 } 15
$+$ to 7 } -8

$$(x+15)(x-8) = 0$$

Either $x + 15 = 0$ or $x - 8 = 0$

$x = -15$ (not suitable)

$\underline{x = 8 \text{ cm}}$