Note Title 17/11/2009

Factorising Quadratic Expressions

A quadratic expression is $ax^2 + bx + c$ where a, b and c are numbers

e.g. $2x^2 + 6x - 9$ $(a=2, b=6, c=-9)$

$x^2 + 5x + 6$ $(a=1 \quad b=5, c=6)$

$4x^2 - 9$ $(a=4 \quad b=0, c=-9)$

Expressions with $a = 1$

To factorize these, we need to find two numbers which

ADD up to b

MULTIPLY to c

e.g. ① $x^2 + 8x + 12$

+ to 8, × to 12 → 6 and 2

$= (x + 6)(x + 2)$

② $x^2 + 11x + 28$

+ to 11, × to 28 → 7 and 4

$= (x + 7)(x + 4)$

If b is −ve and c is +ve, the two numbers we are looking for must both be negative

③ $x^2 - 8x + 15$

add to -8, $\times$ to 15 } -3 and -5

$= (x-3)(x-5)$

④ $x^2 - 17x + 30$

add to -17, $\times$ to 30 } -15 and -2

$= (x-15)(x-2)$

If c is $-ve$, the two numbers we are looking for will be one $+ve$ and one $-ve$.

⑤ $x^2 + 2x - 24$

add to 2, $\times$ to -24 } $+6$ and -4

$= (x+6)(x-4)$

⑥ $x^2 - 2x - 24$

add to -2, $\times$ to -24 } $+4$ and -6

$(x+4)(x-6)$

or $(x-6)(x+4)$

⑦ $x^2 - x - 30$

add to -1, $\times$ to -30 } $+5$ and -6

$(x+5)(x-6)$

Summary of signs :-

$$x^2 + bx + c = (x + \;\;)(x + \;\;)$$

$$x^2 - bx + c = (x - \;\;)(x - \;\;)$$

$$\left.\begin{matrix} x^2 + bx - c \\ x^2 - bx - c \end{matrix}\right\} = (x - \;\;)(x + \;\;)$$

If there are no 'x's (ie, $b = 0$) look out for the "difference of two squares" pattern

$$x^2 - k^2 = (x + k)(x - k)$$

Examples

(8) $x^2 - 9$

$= (x + 3)(x - 3)$

[or write as $x^2 + 0x - 9$

x to -9 } 3 and -3

$+$ to 0]

(9) $x^2 - 49$

$= (x + 7)(x - 7)$

If there is no constant (ie, $c = 0$) we can factorise using a single bracket

(10) $x^2 - 7x = x(x - 7)$

(11) $4x^2 + 10x = 2x(2x + 5)$

Expressions with $a > 1$

- To factorize these we look for two numbers which add to b and multiply to ac.
- Then we split the "bx" term using these numbers
- Then we factorize the first two terms (using a single bracket), and do the same with the last two terms
- The two brackets we get should match, so that we can complete the factorization.

Examples

(1) $2x^2 + 11x + 12$

- $ac = 2 \times 12 = 24$ so we need two numbers which ADD to 11 and MULTIPLY to 24:

8 and 3

[These are NOT the numbers we put in the brackets.]

- Split the $11x$:

$$2x^2 + 8x + 3x + 12$$

- Factorize the first two terms and the last two terms

$$2x(x + 4) + 3(x + 4)$$

- (Check: brackets match) Finish off

$$(2x + 3)(x + 4)$$