

Algebra Problems (Harder)

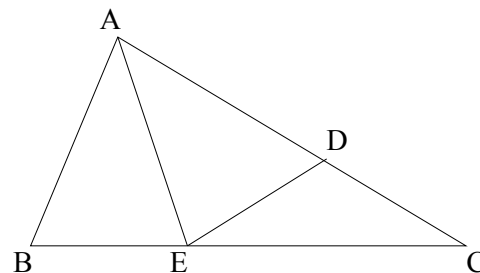
8		
x		6
11		

1) On the left is a magic square which is partly filled in. (In a magic square, the total of the numbers in each row, column and long diagonal is equal.) Find what number is represented by x and complete the magic square.

[Hint: try to express the numbers which go in the blank cells in terms of x , and make an equation]

2) This diagram (which is not drawn accurately) shows a triangle ABC with extra lines AE and DE inside it. If lines AB, AE, DE and DC are all the same length, what is the relationship between angles B and C in the triangle? Make sure you justify your answer.

[Hint: let angle be x° . Now work round the diagram expressing each angle in terms of x .]



3) Alice has been collecting her small change (1p, 2p and 5p coins) to give to charity. When she counted it up, she found that she had exactly 100 coins. There were twice as many 1p coins as 5p coins, and the total amount was £2.24. How many of each type of coin did she have? [Hint: Let the number of 5p coins be x . Express the number of 1p coins and the number of 2p coins in terms of x . Then form an equation.]

4) When Ernie was as old as Bert is now, Bert's age was half of Ernie's present age. When Bert gets to be as old as Ernie is now, the sum of their ages will be 99. How old are Ernie and Bert at the moment?

5) Mr and Mrs Series have had 5 children at regular intervals, so that their ages are all equally spaced. The sum of the ages of the two oldest children is equal to the sum of the ages of the three youngest children. What is the ratio of the ages of the eldest and the youngest child? [Hint: Let the age of the youngest child be x and the gap between each age be g . Express the age of each child in terms of x and g .]

6) I am thinking of three numbers, all different. The sum of all three numbers is 8 times the smallest number. The sum of the largest number and the smallest number is 3 times the middle number. How many times larger than the smallest number is the largest number? Show how you worked this out.

7) Here is a number trick: Think of a three digit number, with all digits different. Now make all the possible two digit numbers which can be formed from the digits of your number (there should be six of them). Add together these six numbers. Now divide by the sum of the digits of your original three digit number. The answer you get is...?

The puzzle is: Prove that you will always get the same answer, no matter what 3 digit number you start with.

[Hint: In algebra, the two digit number formed from the digits x and y is written " $10x + y$ " - not " xy ", which of course means $x \times y$.]

8) Little Jane is playing with the balancing scales from her kitchen. She is putting the plastic animals from her toy zoo onto the scales to see what will balance. She finds that:

one elephant and one giraffe will balance three zebras one giraffe and one zebra will balance four antelopes

one antelope and two zebras will balance one elephant

How many antelopes would it take to balance an elephant?

9) The sequence 3, 7, 10, 17, 27, 44, 71, 115, 186, ... is formed using the rule that, apart from the first two terms, each term is the sum of the two preceding terms. I notice that if I add together the 6th and the 9th terms, I get twice the 8th term ($44 + 186 = 2 \times 115$). Try this with a different sequence formed using the same rule. Prove that it will work for any sequence formed using this rule. Can you generalise this further?

10) A website invites you to give the stories on it a rating from 1 to 5, and displays the average (mean) rating for each story. One story had an average rating of 2.6, but I read it and really liked it, so I gave it a rating of 5. The average rating then changed to 2.75. How many people had rated the story before I did?

11) Jane has been taking a series of tests, and the last one is coming up. She is really keen to achieve an average of 90 over the whole set of tests, but to do this she works out that she needs to score 97 on the last test. In fact she only scores 73 on the last test, so her overall average ends up as 87. How many tests were there in the series?

12) Miss Speed has left it rather late to get to work on time. Driving along, she works out that if she averages 45 miles per hour, she will make it with a minute to spare. If, on the other hand, she only averages 40 mph, she will be one minute late. How far does she have to travel?

13) A mixed packet of nuts containing 1kg of walnuts and 2kg of brazil nuts costs £1.98. A packet containing 4 kg of hazelnuts and 1 kg of walnuts costs £2.48. And for £2.99 you can buy a mixed packet of 3kg of almonds, 1kg of walnuts and 1kg of hazelnuts. How much should you pay for a bag containing 1kg of each of the four types of nut?

14) You are sent to the market by your father with £800, and told to buy 100 animals. When you arrive at the market, you find out that pigs cost £8 each, so it would be easy to follow your father's instructions by buying 100 pigs.

However, you find that there are only 99 pigs for sale. The only other animals there are chickens for £1 each, and cows for £80 each.

If there are enough chickens and cows for you to buy as many as you like, how can you still end up buying 100 animals using exactly £800? Explain how you work this out.

15) Alan throws three darts at a dartboard, and hits three of the numbers from 1 to 20. The total of these three numbers is 31. However, one of the darts lands in the “double” section, and one in the “treble” section, while the third landed in the “single” section, so his overall score for the three darts is 64. Prove that the number of the dart which hit the “treble” was 2 more than the number which hit the “single” section.