

SIMULTANEOUS EQUATIONS

Note Title

14/03/2011

An equation such as $2x + 3 = 12$ has just one solution.

An equation such as

$$2x + 3y = 12$$

has an infinite number of solutions.

e.g. $x = 3, y = 2$

or $x = 9, y = -2$ etc

We can make a table of these solutions and show them as a straight line graph.

If we have two equations of this type

e.g. $2x + 3y = 12$

$$4x - y = 31$$

the solutions of each equation will form a line.

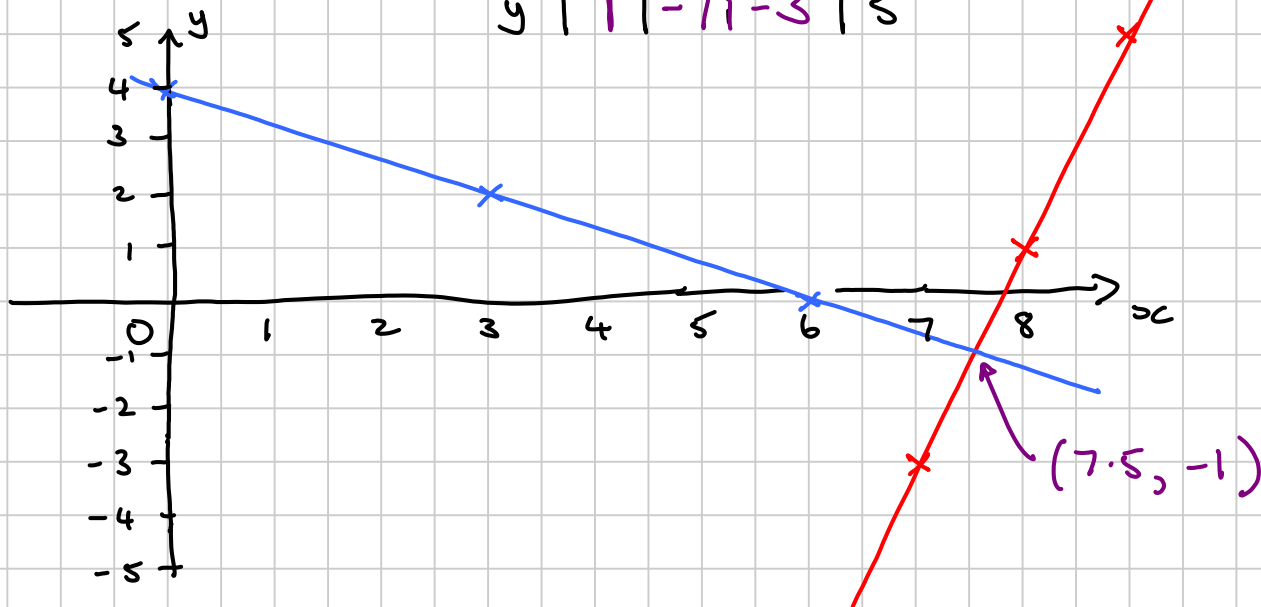
Where the lines cross will be a solution of both equations SIMULTANEOUSLY.

— $2x + 3y = 12$

x	6	0	3
y	0	4	2

— $4x - y = 31$

x	8	6	7	9
y	1	-7	-3	5



Solution is $x = 7.5$, $y = -1$

Elimination Method

This is based on the fact that if we ADD or SUBTRACT two equations, we get another valid equation.

Examples

$$\begin{array}{rcl} \textcircled{1} & 3x + 5y = 16 & \textcircled{1} \\ & 3x + 2y = 10 & \textcircled{2} \end{array}$$

$$\begin{array}{rcl} \textcircled{1} - \textcircled{2} & & \\ & 3y = 6 & \\ & \underline{y = 2} & \end{array}$$

Subst in $\textcircled{1}$: $3x + 10 = 16$

$$\begin{array}{rcl} & (-10) & (-10) \\ & 3x & = 6 \\ & \underline{x = 2} & \end{array}$$

$$\begin{array}{rcl} \textcircled{2} & 5x + 3y = 16 & \textcircled{1} \\ & 2x - 3y = 19 & \textcircled{2} \end{array}$$

$$\begin{array}{rcl} \textcircled{1} + \textcircled{2} & & \\ & 7x & = 35 \\ & \underline{x = 5} & \end{array}$$

Sub in $\textcircled{1}$: $25 + 3y = 16$

$$\begin{array}{rcl} & (-25) & (-25) \\ & 3y & = -9 \end{array}$$

$$3y = -9$$

$$\underline{y = -3}$$

Check in $\textcircled{2}$: $10 - -9 = 19$ ✓

- We can eliminate either x (as in e.g 1) or y (as in e.g 2)
- If the coefficients are equal with the SAME SIGN, SUBTRACT the equations.
- If the coefficients are equal with OPPOSITE SIGNS, ADD the equations.

p 314 Ex 16D Q 9 \rightarrow (WORKING!)

$$\begin{array}{rcl} \textcircled{3} & 5x - 2y = 17 & \textcircled{1} \\ & 3x - 4y = 6 & \textcircled{2} \end{array}$$

$$\begin{array}{rcl} \textcircled{1} \times 2 & 10x - 4y = 34 & \textcircled{3} \\ & 3x - 4y = 6 & \textcircled{2} \end{array}$$

$$\begin{array}{rcl} \textcircled{3} - \textcircled{2} & 7x & = 28 \\ & \underline{x} & = 4 \end{array}$$

Sub in $\textcircled{1}$

$$\begin{array}{rcl} 20 - 2y = 17 & & \\ (-20) & & (-20) \\ -2y = -3 & & \\ (\div -2) & & (\div -2) \\ \underline{y = 1.5} & & \end{array}$$

$$\begin{array}{rcl} \textcircled{4} & 2x + 3y = 17 & \textcircled{1} \\ & 5x - 2y = 14 & \textcircled{2} \end{array}$$

$$\begin{array}{rcl} \textcircled{1} \times 2 & 4x + 6y = 34 & \textcircled{3} \\ \textcircled{2} \times 3 & 15x - 6y = 42 & \textcircled{4} \end{array}$$

$$\begin{array}{rcl} \textcircled{3} + \textcircled{4} & 19x & = 76 \\ & \underline{x} & = 4 \end{array}$$

Sub in $\textcircled{1}$

$$\begin{array}{rcl} 8 + 3y = 17 & & \\ & 3y = 9 & \\ & \underline{y = 3} & \end{array}$$

Problems leading to Sim. Eqns

Example Tickets for the school play are priced at £3 for students and £5 for adults. At the opening night all 200 tickets were sold and the takings were £856. How many tickets were bought by students and how many by adults?

Let x = no of adult tickets
 y = no of student tickets

$$\text{Total no of tickets: } x + y = 200 \quad (1)$$

$$\text{Total takings: } 5x + 3y = 856 \quad (2)$$

$$(1) \times 5 \quad 5x + 5y = 1000 \quad (3)$$

$$(3) - (2) \quad 2y = 144$$
$$y = 72$$

$$\text{Sub in (1)} \quad x + 72 = 200$$
$$x = 128$$

128 adult tickets and 72 student tickets