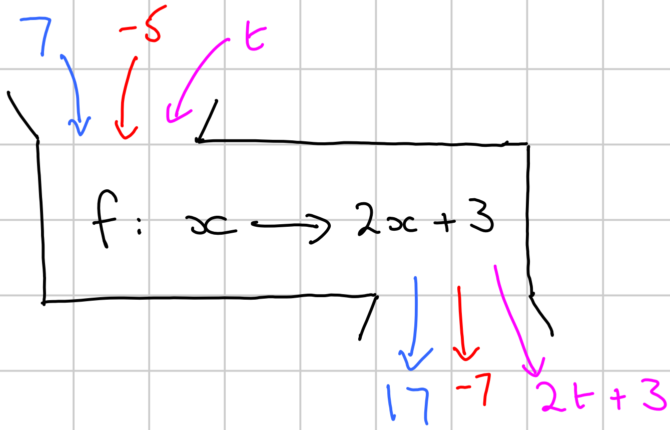
Note Title 15/10/2008

FUNCTIONS

A function is a rule or process which we can do to a number to obtain another number.

We can visualise a function as a machine

e.g.



We can write the result of putting 7 into the function as

$$f(7) = 17$$

More examples :—

① $f(10) = 23$

② $f(t+4) = 2(t+4) + 3$
$= 2t + 8 + 3$
$= 2t + 11$

Let $g : x \longrightarrow x^2$ and $h : x \longrightarrow \frac{8}{x}$

③ $g(5) = 25$

④ $g(-3) = 9$

⑤ $h(2) = 4$

Domain and Range

The DOMAIN of a function is the set of numbers which are allowed to be 'put in' to the function.

The RANGE is the set of numbers which the function can produce from the given domain.

Example ① If the domain of f is $5 \leq x \leq 10$, what is the range of f?

Answer : range is $13 \leq f(x) \leq 23$

② If the domain of g is any number what is its range?

Answer : range of g is $g(x) \geq 0$

Sometimes it is necessary to exclude certain number(s) from the domain, e.g.

③ Which number MUST be excluded from the domain of h?

Answer : 0

Working backwards

Example Find the value of x if

(a) $f(x) = 24$

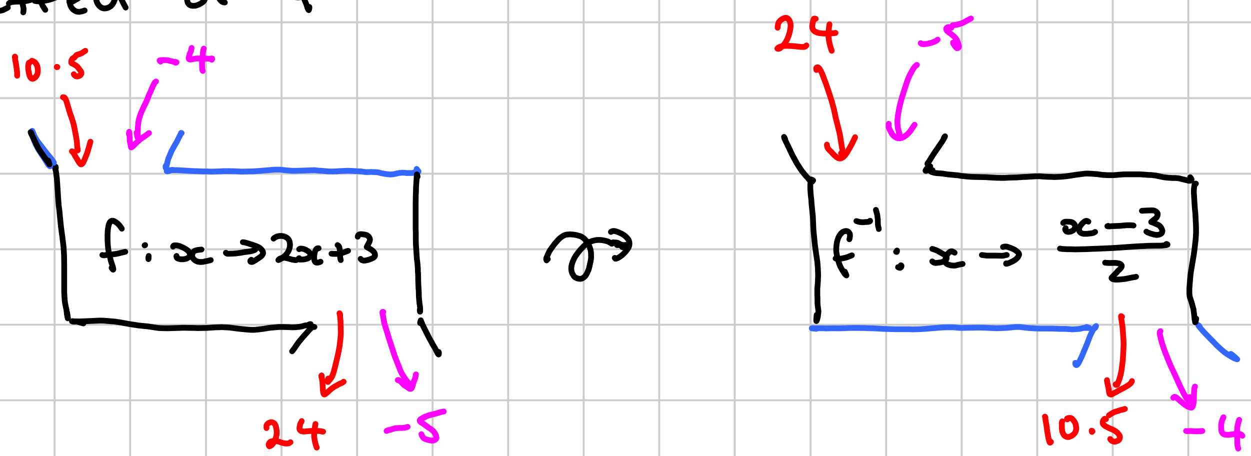
This is the same as solving an equation

$$2x + 3 = 24$$
$(-3) \qquad (-3)$
$$2x = 21$$
$(\div 2) \qquad (\div 2)$
$$x = \underline{\underline{10.5}}$$

(b)
$$h(x) = 24$$
$$\frac{8}{x} = 24$$
$(\times x) \qquad (\times x)$
$$8 = 24x$$
$(\div 24) \qquad (\div 24)$
$$\frac{8}{24} = x$$
$$\underline{\underline{x = \frac{1}{3}}}$$

Inverse Functions

The inverse of a function f is written f^{-1} and means the function which 'undoes' the effect of f



The method for finding an inverse is as follows :—

- write the function as "$y = f(x)$"
- CHANGE THE SUBJECT to "$x = f^{-1}(y)$"
- rewrite using x instead of y

e.g.

$$y = 2x + 3$$
$$(-3) \qquad (-3)$$
$$y - 3 = 2x$$
$$(\div 2) \qquad (\div 2)$$
$$\frac{y-3}{2} = x$$

so $f^{-1} : x \longrightarrow \frac{x-3}{2}$

Harder examples:

① If $f : x \rightarrow 3x^2 + 5$, find f^{-1}.

$$y = 3x^2 + 5$$
$$(-5) \qquad (-5)$$
$$y - 5 = 3x^2$$
$$(\div 3) \qquad (\div 3)$$
$$\frac{y-5}{3} = x^2$$
$$(\sqrt{\ }) \qquad (\sqrt{\ })$$
$$\sqrt{\frac{y-5}{3}} = x$$

So $f^{-1} : x \longrightarrow \sqrt{\frac{x-5}{3}}$

② The function f is defined by

$$f : x \rightarrow \frac{x+3}{x-2} \quad (\text{domain: } x \neq 2)$$

Find the inverse function f^{-1}, stating its domain.

Write $\quad y = \dfrac{x+3}{x-2}$

$(\times (x-2))$ $\quad y(x-2) = x+3$

$$xy - 2y = x + 3$$

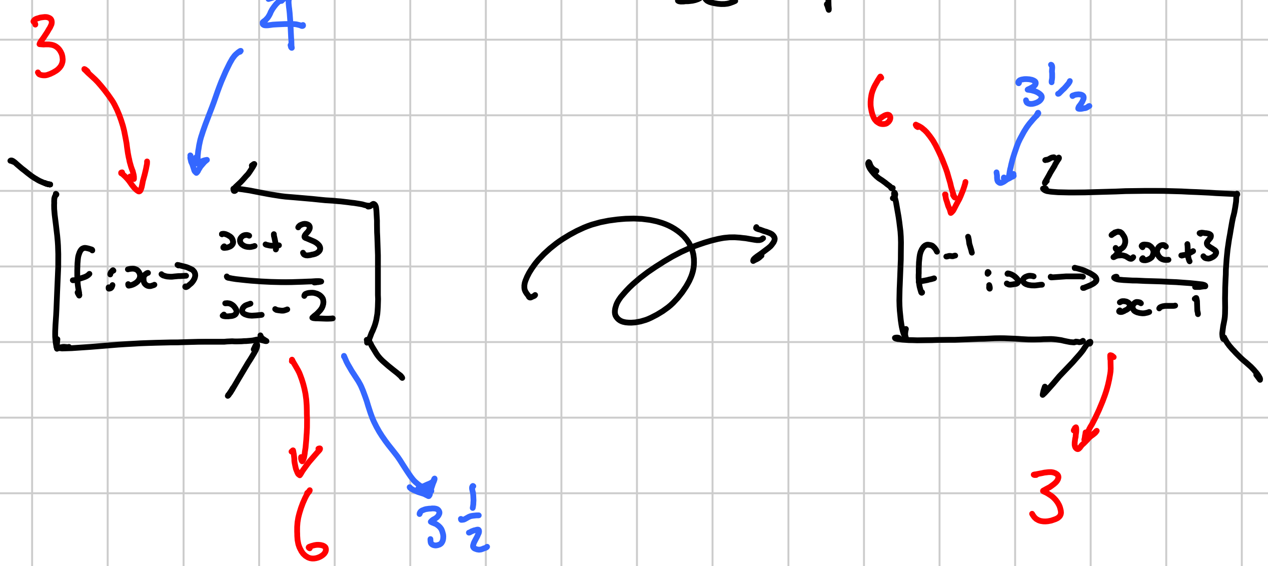
(get all x's on one side) $\quad (-x) \; (+2y) \qquad (-x) \; (+2y)$

$$xy - x = 2y + 3$$

(factorize so that x appears once) $\quad x(y-1) = 2y+3$

$$x = \frac{2y+3}{y-1}$$

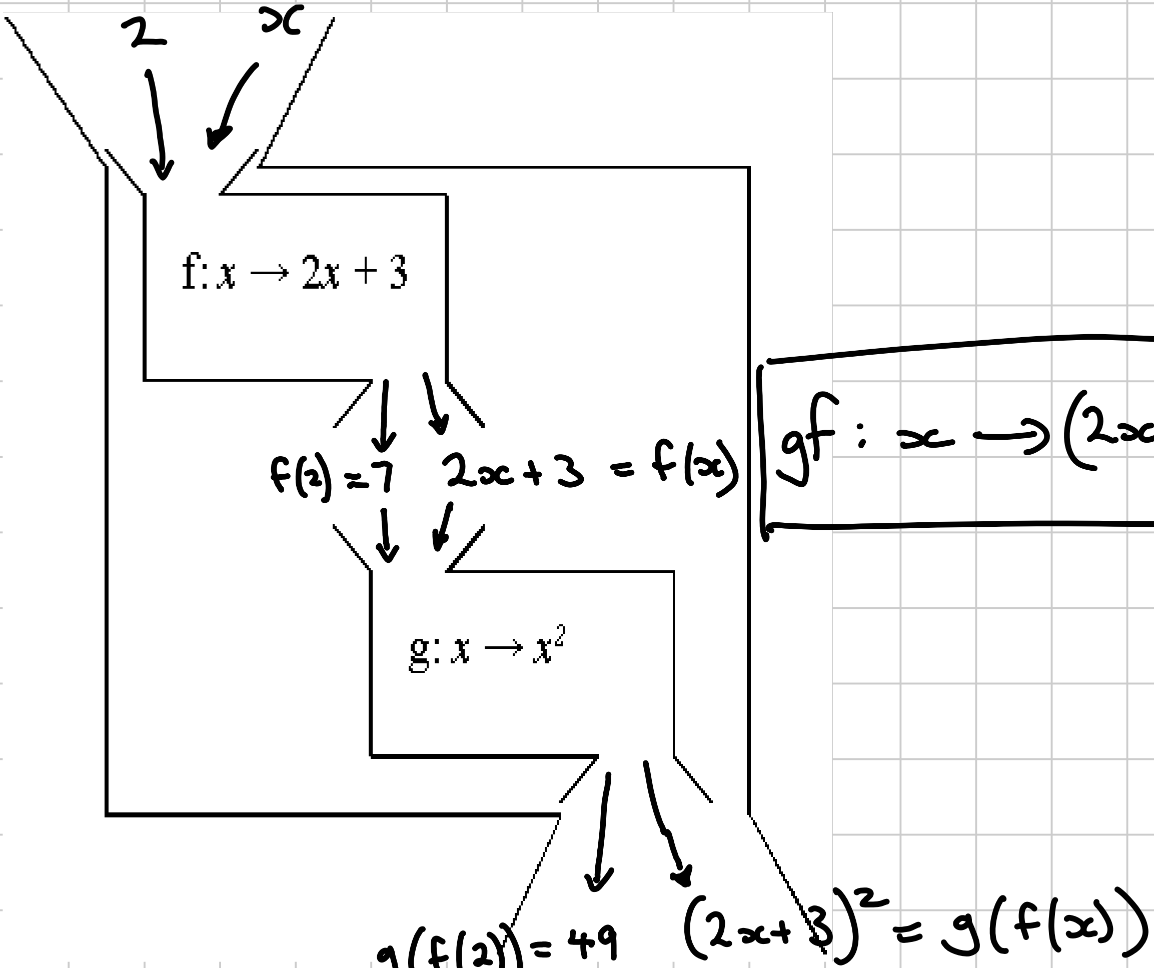
So $f^{-1} : x \rightarrow \dfrac{2x+3}{x-1}$ (domain $x \neq 1$)



Composite Functions

These are formed by feeding the output of one function into a second function.

Example :-



$\boxed{gf : x \to (2x+3)^2}$

The composite function works out $g(f(x))$.
ie, start with x, do f and get a result, then do g to that result to get the final output.

We can write this in a slightly shorter form as $gf(x)$

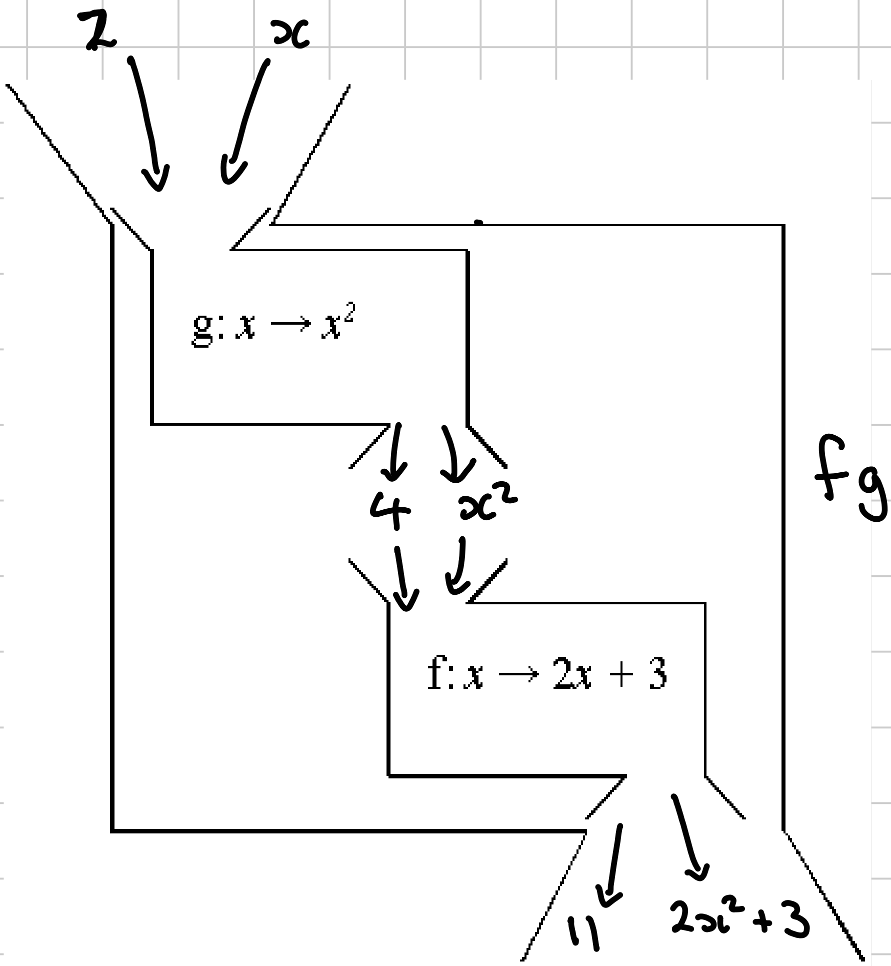
Note that this means <u>DO f, THEN DO g</u>

To wite gf as a single function, find the output when 'x' is fed into the function.

From above we see that the composite function gf is written

$$gf : x \rightarrow (2x+3)^2$$

<u>NB</u> This is different to fg, which is as follows :-



$$fg : x \rightarrow 2x^2 + 3$$

Example (without diagrams!)

If $f : x \longrightarrow 3x - 2$

and $g : x \longrightarrow \frac{1}{x+5}$

(a) Find $gf(5)$.

$$g(f(5)) = g(13)$$
$$= \underline{\underline{\frac{1}{18}}}$$

(b) Find $fg(5)$

$$f(g(5)) = f\left(\frac{1}{10}\right)$$
$$= 3 \times \frac{1}{10} - 2$$
$$= \underline{\underline{-1.7}}$$

(c) Find gf as a function

$$gf(x) = g(3x-2)$$
$$= \frac{1}{3x-2+5}$$
$$= \frac{1}{3x+3}$$

So $gf : x \longrightarrow \frac{1}{3x+3}$