PROPORTIONALITY

The sign $\propto$ means "is proportional to"
A statement such as "$y \propto x$" can always be replaced by "$y = kx$".
The constant $k$ is called a "constant of proportionality".

Example: An object is dropped from an aircraft.
The distance ($s$) which it has fallen is proportional to the square of the time ($t$) for which it has been falling. After 4 seconds, it has fallen 80 m.

(a) Find a formula for $s$ in terms of $t$

Write in symbols: $s \propto t^2$
Convert to a formula: $s = kt^2$

To find $k$, substitute in the given values

\[ 80 = k \times 16 \]
\[ k = 5 \]

Then put this value of $k$ into the formula

\[ s = 5t^2 \]

(b) How far will the object fall in 8 seconds?

Substitute $t = 8$, $s = 5 \times 8^2 = 320$ m

(c) How long would it take for the object to fall 500 m?

Substitute $s = 500$:

\[ 500 = 5t^2 \]
\[ (\div 5) \quad (\div 5) \]
\[ 100 = t^2 \]
\[ 10 = t \]

It would take 10 seconds.
Inverse Proportionality

If \( y \) is inversely proportional to \( x \), then doubling \( x \) causes \( y \) to be halved.

We write this as \( y \propto \frac{1}{x} \)

which turns into \( y = \frac{k}{x} \)

Example: The time \( (T) \) to do a job is inversely proportional to the number of people \((n)\) working on it. With 3 people the job will take 6 hours. How long will it take with 10 people?

Write in symbols \( T \propto \frac{1}{n} \)

Write with "\( = \) and \( k \)" \( T = \frac{k}{n} \)

Use the numbers given to find \( k \):

\[ 6 = \frac{k}{3} \]

\( \times 3 \) \( \times 3 \)

\[ 18 = k \]

Write the formula replacing \( k \) by the number \( T = \frac{18}{n} \)

Now we can use this formula to answer questions:

If \( n = 10 \), \( T = \frac{18}{10} = 1.8 \) hours
(b) How many people would be needed to do the job in 1 hour?

If \( T = 1 \),

\[
1 = \frac{18}{n}
\]

\[
(\times n) \quad (\times n)
\]

\[ n = 18 \text{ people} \]