SIMULTANEOUS EQUATIONS

An equation such as \( 2x + 3 = 12 \) has just one solution.

An equation such as

\[
2x + 3y = 12
\]

has an infinite number of solutions.

e.g. \( x = 3, \ y = 2 \)

or \( x = 9, \ y = -2 \) etc

We can make a table of these solutions and show them as a straight line graph.

If we have two equations of this type

\[
\begin{align*}
2x + 3y &= 12 \\
4x - y &= 31
\end{align*}
\]

the solutions of each equation will form a line. Where the lines cross will be a solution of both equations SIMULTANEOUSLY.

\[
\begin{array}{c|ccc}
2x + 3y &= 12 & \quad x & \quad 6 & \quad 0 & \quad 3 \\
y & \quad 0 & \quad 4 & \quad 2
\end{array}
\]

\[
\begin{array}{c|ccc}
4x - y &= 31 & \quad x & \quad 8 & \quad 6 & \quad 7 & \quad 9 \\
y & \quad 1 & \quad -7 & \quad -3 & \quad 5
\end{array}
\]
Solution is \( x = -7.5, \ y = -1 \)

**Elimination Method**

This is based on the fact that if we ADD or SUBTRACT two equations, we get another valid equation.

**Examples**

1. \[
\begin{align*}
3x + 5y &= 16 \\
3x + 2y &= 10
\end{align*}
\]

0 - 2

\[
\begin{align*}
3y &= 6 \\
y &= 2
\end{align*}
\]

Subst in 0:

\[
\begin{align*}
3x + 10 &= 16 \\
(-10) &\quad (-10)
\end{align*}
\]

\[
\begin{align*}
3x &= 6 \\
x &= 2
\end{align*}
\]

2. \[
\begin{align*}
5x + 3y &= 16 \\
2x - 3y &= 19
\end{align*}
\]

1 + 2

\[
\begin{align*}
7x &= 35 \\
x &= 5
\end{align*}
\]

Subst in 1:

\[
\begin{align*}
25 + 3y &= 16 \\
(25) &\quad (25)
\end{align*}
\]

\[
\begin{align*}
3y &= -9 \\
y &= -3
\end{align*}
\]

Check in 2:

\[
10 - 9 = 19 \checkmark
\]
• We can eliminate either \( x \) (as in e.g. 1) or \( y \) (as in e.g. 2)

• If the coefficients are equal with the **SAME SIGN**, SUBTRACT the equations.
• If the coefficients are equal with opposite signs, ADD the equations.

\[ \begin{align*}
3 \quad 5x - 2y &= 17 \\
3x - 4y &= 6 \\
\times 2 & \quad 10x - 4y = 34 \\
3x - 4y &= 6 \\
\hline
3 - 3 & \quad 7x = 28 \\
\hline
\end{align*} \]

\[ x = 4 \]

Sub in 1
\[ \begin{align*}
20 - 2y &= 17 \\
(-20) & \quad (-20) \\
-2y &= -3 \\
\div (-2) & \quad (\div (-2)) \\
y &= 1.5
\end{align*} \]

\[ \]

\[ \begin{align*}
4 \quad 2x + 3y &= 17 \\
5x - 2y &= 14 \\
\times 2 & \quad 4x + 6y = 34 \\
\times 3 & \quad 15x - 6y = 42 \\
\hline
3 + 4 & \quad 19x = 76 \\
\hline
x &= 4
\end{align*} \]

Sub in 1
\[ \]
\[ \begin{align*}
8 + 3y &= 17 \\
3y &= 9 \\
y &= 3
\end{align*} \]

\[ \]
Problems leading to Sim. Eqns

Example Tickets for the school play are priced at £3 for students and £5 for adults. At the opening night all 200 tickets are sold and the takings are £856. How many tickets were bought by students and how many by adults?

Let \( x \) = no of adult tickets

\( y \) = no of student tickets

Total no of tickets: \( x + y = 200 \) \( \quad (1) \)

Total takings: \( 5x + 3y = 856 \) \( \quad (2) \)

\( 1 \times 5 \)

\( 5x + 5y = 1000 \) \( \quad (3) \)

\( 3 - 2 \)

\( 2y = 144 \)

\( y = 72 \)

Sub in \( (1) \)

\( x + 72 = 200 \)

\( x = 128 \)

128 adult tickets and 72 student tickets