Note Title 11/02/2009

Converting Recurring Decimals to Fractions

Examples

① $0.\dot{0}2\dot{7}$ or $0.\overline{027}$

$x = 0.027027027\ldots\ldots$ ①

$1000x = 27.027027027\ldots\ldots$ ②

[Note: because the recurring part is 3 digits long we need to multiply by 1000 to make the recurring digits line up again.]

Subtract ② − ①: $999x = 27$

$x = \frac{27}{999} \overset{\div 9}{=} \frac{3}{111} \overset{\div 3}{=} \frac{1}{37}$

② $0.0\dot{2}\dot{7}$ or $0.0\overline{27}$

$x = 0.027272727\ldots$ ①

$100x = 2.727272727\ldots$ ②

② − ①: $99x = 2.7$

$x = \frac{2.7}{99} \overset{\times 10}{=} \frac{27}{990} \overset{\div 9}{=} \frac{3}{110}$

③ $0.02\dot{7}$

$x = 0.027777\ldots$

$10x = 0.277777\ldots$

② − ① : $9x = 0.25$

$x = \frac{0.25}{9} = \frac{25}{900} = \frac{1}{36}$

(×100 on both numerator and denominator; ÷25 on both numerator and denominator)

1	digit in	recurring part	⇒	× by 10
2	digits in	recurring part	⇒	× by 100
3	digits in	recurring part	⇒	× by 1000