RULES OF INDICES

Note Title 20/04/2009

① To MULTIPLY powers of the SAME NUMBER, ADD the INDICES

e.g $4^3 \times 4^5 = 4^{3+5} = 4^8$

- must be the same number : $4^3 \times 7^5$ can't be simplified
- don't multiply the numbers : $4^3 \times 4^5$ is NOT 16^8 !

② To DIVIDE powers of the same number, SUBTRACT the indices

e.g $\frac{4^5}{4^3} = 4^{5-3} = 4^2$

③ To find a POWER of a POWER, MULTIPLY the indices

e.g $(4^3)^5 = 4^{3\times5} = 4^{15}$

④ $x^0 = 1$ whatever number x is

e.g. $4^0 = 1$

$17^0 = 1$

⑤ $x^{-n} = \frac{1}{x^n}$ ie/ a <u>negative</u> index means the <u>reciprocal</u>

e.g. $4^{-2} = \frac{1}{4^2}$ or $\frac{1}{16}$

$2^{-3} = \frac{1}{2^3}$ or $\frac{1}{8}$

$7^{-1} = \frac{1}{7^1}$ or $\frac{1}{7}$

More examples

① $5^{-7} \times 5^{3} = 5^{-7+3} = 5^{-4}$

② $\dfrac{7^4}{7^{-5}} = 7^{4--5} = 7^{9}$

③ $(4^{-3})^5 = 4^{-3\times 5} = 4^{-15}$

④ $3x^3 \times 4x^4 = 12x^7$

⑤ $(2x^2)^3 =$ (everything in the bracket is cubed)

$= 2^3 \; (x^2)^3$

$= 8x^6$

⑥ 6^{-2} as a fraction is $\dfrac{1}{6^2} = \dfrac{1}{36}$

Further examples

① $\dfrac{3^5 \times 3^7}{3^{10}} = \dfrac{3^{12}}{3^{10}} = 3^2 = 9$

② $5^{-3} = \dfrac{1}{5^3} = \dfrac{1}{125}$

③ $7^{-1} = \dfrac{1}{7^1} = \dfrac{1}{7}$

④ $6^5 \times 6 = 6^5 \times 6^1 = 6^6$

⑤ $\dfrac{2^5}{2^{-2}} = 2^{5--2} = 2^7 \quad (= 128)$

Explanation of the rules

Consider $\frac{2^3}{2^3}$:-

- By the rule this is $2^{3-3} = 2^0$
- By ordinary arithmetic this is $\frac{8}{8} = 1$

We conclude that 2^0 must equal 1

So $\boxed{x^0 = 1}$

Now consider $\frac{2^3}{2^5}$:—

- By the rule this is $2^{3-5} = 2^{-2}$
- By ordinary arithmetic this is $\frac{\cancel{2}^1 \times \cancel{2}^1 \times \cancel{2}^1}{\cancel{2}_1 \times \cancel{2}_1 \times \cancel{2}_1 \times 2 \times 2} = \frac{1}{4}$

So we conclude that $2^{-2} = \frac{1}{4}$

Similarly, $3^{-2} = \frac{1}{9}$, $5^{-2} = \frac{1}{25}$

Generalizing, $\boxed{x^{-2} = \frac{1}{x^2}}$

Going further, $\boxed{x^{-n} = \frac{1}{x^n}}$