If we are told that the length of a building is 12 m (to the nearest m) then we know that

\[11.5 \text{ m} \leq \text{length} < 12.5\]

This is called the lower bound of the length.

Note that the number can be equal to the lower bound but must be less than the upper bound.

More examples:

1. \(x = 1.47\) (to 2 dp)
   
   Possible values before rounding:
   
   \[1.465 \leq x < 1.475\]

2. \(x = 0.531\) (to 3 sf)
   
   \[0.5305 \leq x < 0.5315\]

3. \(x = 23800\) (to nearest 100)
   
   \[23750 \leq x < 23850\]

Another way we may see these written is:

1. \(x = 1.47 \pm 0.005\)
   
   "PLUS OR MINUS"

2. \(x = 0.531 \pm 0.0005\)

3. \(x = 23800 \pm 50\)
Calculations with Upper and Lower Bounds

- Find the UB or LB of each value involved FIRST, then do the calculation.

**Addition and Multiplication:**

\[
\begin{align*}
\text{LB of } x + y &= \text{LB of } x + \text{LB of } y \\
\text{LB of } xy &= \text{LB of } x \times \text{LB of } y \\
&\text{(same for UBs)}
\end{align*}
\]

- e.g. A field is a rectangle with a length of 240m and width of 160m (both to 2sf).

(a) Find the lower bound of the area of the field.

\[
\begin{align*}
\text{LB of length} &= 235m \\
\text{LB of width} &= 155m
\end{align*}
\]

\[
\text{LB of area} = 235 \times 155 = 36425 \text{ m}^2
\]

(b) Find the upper bound of perimeter.

\[
\begin{align*}
\text{UB of perimeter} &= (245 + 165) \times 2 \\
&= 820 \text{ m}
\end{align*}
\]

**Subtraction and Division:**

\[
\begin{align*}
\text{LB of } \frac{x - y}{y} &= \text{LB of } x - \text{UB of } y \\
\text{UB of } \frac{x}{y} &= \text{UB of } x - \text{LB of } y
\end{align*}
\]

- e.g. 1) Luddy is 175cm tall and Tash is 155cm tall. (both to the nearest cm)

Find the lower bound of the difference in height.

\[
\begin{align*}
\text{LB of difference} &= 174.5 - 155.5 \\
&= 19 \text{ cm}
\end{align*}
\]
2. The 100m track is marked out to the nearest metre. Annabelle runs the 100m in 13.5 s (to 1d.p.). Find the UB of Annabelle’s speed.

\[
\text{UB of speed} = \frac{\text{UB of distance}}{\text{UB of time}}
\]

\[
= \frac{100.5 \text{m}}{13.45 \text{s}}
\]

\[
= 7.47 \text{m/s}
\]