

UPPER AND LOWER BOUNDS

If we are told that the length of a building is 12m (to the nearest m) then we know that

$$11.5\text{m} \leq \text{length} < 12.5$$

this is called the LOWER BOUND of the length

this is called the UPPER BOUND

Note that the number can be equal to the lower bound but must be less than the upper bound.

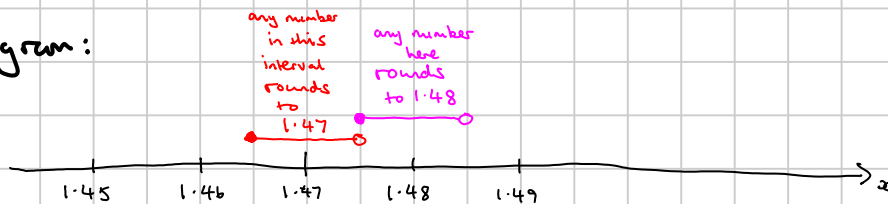
More examples

① $x = 1.47$ (to 2 dp)

Possible values before rounding

$$1.465 \leq x < 1.475$$

On a diagram:



(note that a filled in circle indicates that the number can be equal to that bound, but an empty circle means it cannot be equal to that bound)

② $x = 0.531$ (to 3 sf)

$$0.5305 \leq x < 0.5315$$

③ $x = 23800$ (to nearest 100)

$$23750 \leq x < 23850$$

Another way we may see these written is :-

① $x = 1.47 \pm 0.005$

“PLUS OR MINUS”

② $x = 0.531 \pm 0.0005$

③ $x = 23800 \pm 50$

Calculations with Upper and Lower Bounds

- Find the UB or LB of each value involved FIRST, then do the calculation.

Addition and Multiplication:

$$\begin{aligned} \text{LB of } x + y &= \text{LB of } x + \text{LB of } y \\ \text{LB of } xy &= \text{LB of } x \times \text{LB of } y \\ (\text{same for UBs}) \end{aligned}$$

e.g. A field is a rectangle with a length of 240m and width of 160m (both to 2sf).

- (a) Find the lower bound of the area of the field.

$$\text{LB of length} = 235\text{m}$$

$$\text{LB of width} = 155\text{m}$$

$$\text{LB of area} = 235 \times 155 = \underline{36425\text{m}^2}$$

- (b) Find the upper bound of perimeter.

$$\begin{aligned} \text{UB of perimeter} &= (245 + 165) \times 2 \\ &= \underline{820\text{m}} \end{aligned}$$

Subtraction and Division:

$$\text{LB of } x - y = \text{LB of } x - \text{UB of } y$$

$$\text{LB of } \frac{x}{y} = \frac{\text{LB of } x}{\text{UB of } y}$$

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e.g. (i) Luddy is 175cm tall and Tash is 155cm tall.
(both to the nearest cm)

Find the lower bound of the difference in height.

$$\begin{aligned} \text{LB of difference} &= 174.5 - 155.5 \\ &= \underline{\underline{19\text{cm}}} \end{aligned}$$

② The 100m track is marked out to the nearest metre. Annabelle runs the 100m in 13.5s (to 1dp). Find the UB of Annabelle's speed.

$$\text{UB of speed} = \frac{\text{UB of distance}}{\text{LB of time}}$$

$$= \frac{100.5 \text{ m}}{13.45 \text{ s}}$$

$$= \underline{\underline{7.47 \text{ m/s}}}$$

$$\left(\begin{array}{r} \text{LB} \\ 99.5 \\ \hline 13.55 \\ \hline = 7.34 \text{ m/s} \end{array} \right)$$