PERCENTAGES

To convert a fraction or a decimal to a percentage, multiply by 100.

To convert a percentage to a fraction or a decimal, divide by 100.

Examples

1. Convert each of the following to a percentage

   (a) \(0.36\)

      \[0.36 \times 100 = 36\%\]

   (b) \(0.074\)

      \[0.074 \times 100 = 7.4\%\]

   (c) \(\frac{9}{20}\)

      \[
      \frac{9}{20} \times \frac{100}{100} = 45\%
      
      \]

   (d) \(\frac{11}{40}\)

      \[
      \frac{11}{40} \times \frac{100}{100} = \frac{55}{2} = 27.5\% \]

2. Convert to a decimal

   (a) \(12.5\%\)

      \[12.5 \div 100 = 0.125\]

   (b) \(130\%\)

      \[130 \div 100 = 1.3\]

3. Convert to a fraction

   (a) \(48\%\)

      \[
      \frac{12}{100} = \frac{12}{25}
      
      \]

   (b) \(13\frac{1}{3}\%\)

      \[
      \frac{40}{300} = \frac{4}{30} = \frac{2}{15}
      
      \]
4) Expressing one number as a percentage of another.

E.g. Out of 640 students in a school, 96 come to school by bike.

What percentage of the students come by bike?

\[
\frac{96}{640} \times \frac{5}{1} = 15\%
\]

5) Finding a percentage of a number.

- In this situation, "OF" means X.
- We can change the percentage to a fraction or a decimal, whichever we prefer.

E.g. 1) Out of 650 students in a school, 24% come to school by car.

How many students come by car?

i.e. find 24% of 650

\[
\frac{24}{100} \times \frac{13}{2} = \frac{156}{100}
\]

[OR \(0.24 \times 650 = 156\)]

2) The price of a computer has increased by 12%.

If it was £425, what is it now?

12% of 425 = 0.12 \times 425

= £51

So new price = 425 + 51 = £476

[But we will see a better way to do this next lesson.]
Percentage Increase and Decrease

To increase or decrease an amount by a certain percentage, we multiply by a MULTIPLYING FACTOR.

Examples

1. The price of a computer has increased by 12%. If it was £425 before, what is it now?

   Original price = 100% so new price = 112%

   Find 112% of £425

   \[ 1.12 \times 425 = £476 \]

   [To increase by 12%, the multiplying factor is 1.12]

2. The population of a town has decreased by 28%. If it was 115000, what is it now?

   Original pop = 100% so New pop = 72%

   Find 72% of 115000

   \[ 0.72 \times 115000 = 82800 \text{ people} \]

3. Write down the multiplying factor to:

   (a) Increase by 8% \[ 1.08 \]
   (b) Decrease by 5\(\frac{1}{2}\)% \[ 0.945 \]
   (c) Increase by 17\(\frac{1}{2}\)% \[ 1.175 \]
Finding what percentage \( A \) is of \( B \)

To do this, we write

\[
\frac{A}{B} \times \frac{100}{1}
\]

Examples

1. Out of 540 pupils in a school, 144 come to school by car. What percentage come by car?

\[
\frac{144}{540} \times \frac{100}{1} = \frac{80}{3} \approx 26.67\%
\]

2. Alice receives £7.20 pocket money per week, of which she spends £2.70 on sweets. What percentage does she spend on sweets?

\[
\frac{2.70}{7.20} \times \frac{100}{1} = 37.5\%
\]

Profit and Loss

To work out a percentage profit or loss we calculate

\[
\text{% profit} = \frac{\text{actual profit}}{\text{original (cost) price}} \times \frac{100}{1}
\]

Examples

1. I bought a Beatles LP for £5 in 1969, and sold it for £20 last year. What percentage profit did I make?

Actual profit = £20 - £5 = £15

\[
\text{Percentage profit} = \frac{15}{5} \times \frac{100}{1} = 300\%
\]
(2) I bought a car for £8000. 3 years later I sold it for £1800. What was my percentage loss?

Actual loss = 8000 - 1800 = £6200

\[
\text{Percentage loss} = \frac{6200}{8000} \times 100 = 77.5\%
\]

Finding the original amount

Examples

1. A shop sells a camera for £132, making a profit of 10%. How much does the shop pay for the camera?

Original price = \( x \)  

Multiplying factor = 1.1

\[ 1.1 \times x = 132 \]

\[
x = \frac{132}{1.1} = £120
\]

2. The population of a village fell by 15% between 2000 and 2010. In 2010 it was 5270. What was it in 2000?

Population in 2000 = \( x \)  

Multiplying factor = 0.85

\[ 0.85 \times x = 5270 \]

\[
x = \frac{5270}{0.85} = 6200
\]
Compound Percentages

Examples

1. I invest £10000 in a fixed-rate bond for 6 years at 5% per year interest. How much will I receive back after 6 years?
   Multiplying factor to increase by 5% is 1.05
   Answer is £10000 x 1.05^6 = £13400.96

2. A car depreciates (goes down in value) by 30% in its first year, and 15% in each of the next 4 years. I buy a new car for £12000. How much is it worth after 5 years?
   \[12000 \times 0.7 \times 0.85^4 = £4385\]

3. The population of a town increases by 30%, then decreases by 25%. What percentage has the population increased by overall?
   \[P \times 1.3 \times 0.75 = P \times 0.975\]
   (Trick question!) Population has decreased by 2.5%.