Comparing Fractions

1   Without using a calculator and without doing any written calculation, decide which of the two numbers $\frac{13}{18}$ and $\frac{11}{15}$ is the larger.

Explain clearly the reasons for your answer.

The following questions explore six different, but related, ways of deciding which of two fractions is the larger. As you work through the questions:

- Check whether your *answer* to question 1 was correct.
- Check whether the *reason* you gave was correct, or how it could have been improved.

To begin with, you are free to use a calculator.

2   **Method One: Subtraction**

Work out one of these subtractions: $\frac{13}{18} - \frac{11}{15}$ or $\frac{11}{15} - \frac{13}{18}$.

How does your answer tell you which of these fractions is the larger?

3   **Method Two: Fraction of a Suitably Chosen Quantity *N***

Choose an integer N that is divisible by both 18 and 15.

Work out $\frac{13}{18}$ of N.

Work out $\frac{11}{15}$ of N.

How do your two answers tell you which of these two fractions is the larger one?

4 | **Method Three: Equivalent Fractions**

Produce a list of fractions equivalent to $\frac{11}{15}$:

$$\frac{11}{15} = \frac{22}{30} = \frac{33}{45} = \frac{44}{60} = \cdots$$

Produce a list of fractions equivalent to $\frac{13}{18}$:

$$\frac{13}{18} = \frac{26}{36} = \frac{39}{54} = \frac{52}{72} = \cdots$$

Extend both lists until you find two fractions with the same denominator: one in each list. Write down these two fractions.

Compare the numerators of these two fractions.

How does this tell you which of the two fractions $\frac{13}{18}$ and $\frac{11}{15}$ is the larger one?

5 | **Method Four: Comparing Decimal Approximations**

Evaluate each fraction as a decimal.

How do your two answers tell you which of the fractions is the larger?

6 Look carefully at each of the above four methods. Make sure that you understand how to carry out each method *without using a calculator*.

7 $\frac{2}{3}$ and $\frac{4}{5}$ are familiar fractions.

(a) Work out in your head which is the larger.

(b) Which of the first four methods would you use to check your answer using a calculator? Why?

(c)   Which method would you use without a calculator? Why?

Now try to do *without a calculator*.

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8 | **Method Five: Cross-Multiplication**

Suppose you are given two fractions (such as $\frac{2}{3}$ and $\frac{4}{5}$).

First multiply the numerator of $\frac{2}{3}$ by the denominator of $\frac{4}{5}$.

Then multiply the denominator of $\frac{2}{3}$ by the numerator of $\frac{4}{5}$.

Compare your two answers.

How does this tell you which of the two fractions is the larger? Explain.

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9 | **Method Six: Equivalent Fractions (Again)**

Method Three is often called the *method of finding a common denominator*. We work out which of (say) $\frac{1}{3}$ and $\frac{2}{7}$ is bigger by reasoning as follows: $\frac{1}{3} = \frac{7}{21}$ and $\frac{2}{7} = \frac{6}{21}$. $\frac{7}{21} > \frac{6}{21}$. So $\frac{1}{3} > \frac{2}{7}$.

The real-life version of this problem says *Which is better, one bottle for three or two bottles for seven?* It suggests another solution: *One bottle for three is equivalent to getting two bottles for six. But getting two bottles for six is better than getting two bottles for seven. So $\frac{1}{3} = \frac{2}{6} > \frac{2}{7}$.*

In other words, we can find a *common numerator*, instead of a common denominator.

Use this method to compare $\frac{11}{15}$ and $\frac{13}{18}$.

Puzzles

10 Which of the fractions

$$\frac{10001}{10002} \quad \text{and} \quad \frac{100001}{100002}$$

is the larger?

(Hint: Both fractions are less than 1. How much less than 1 are they?)

11 Which of the fractions

$$\frac{12345}{54321} \quad \text{and} \quad \frac{12346}{54322}$$

is the larger?

12 Immanuel has a $\frac{2}{3}$-metre-long piece of string. He needs a piece of length $\frac{1}{2}$ metre. Unfortunately he does not have a ruler. How can Immanuel get a piece of string that is exactly $\frac{1}{2}$ metre long without using a ruler?