Solving Differential Equations by Substitution Methods

1) Use the substitution y = vx (where v is a function of x) to solve the equation

$$x^2\frac{dy}{dx}=x^2+xy+y^2$$

given that y=0 when x=2.

2) Show that the substitution $z=\frac{1}{y^2}$ transforms the equation

$$2\frac{dy}{dx}-y=2y^3e^x$$

into the equation

$$\frac{dz}{dx}+z=-2e^x$$

and hence find the general solution of the original equation

3) Use the substitution $w=\frac{dy}{dx}$ to eliminate y from the equation

$$x\frac{d^2y}{dx^2}+2\frac{dy}{dx}=x\ln x$$

and hence find the general solution of this equation.

4) Use the substitution $w=\frac{dy}{dx}$ to eliminate x from the equation

$$y^3\frac{d^2y}{dx^2}+1=0$$

Hence find the solution of this equation for which y=1 and $\frac{dy}{dx}=2$ when x=0.

5) Use the substitution $y=x^2+z$ (where z is a function of x) to find the general solution of the equation

$$(1-x^2)\frac{dy}{dx}+xy=2\text{x}-x^3$$

6) Use the substitution $x=e^t$ to find the general solution of the equation

$$x^2\frac{d^2y}{dx^2}-3x\frac{dy}{dx}+3y=x^2$$

7) Use the substitution y = z - x to find the solution of

$$(x+y)\frac{dy}{dx}=x+y-2$$

for which y=2 when x=2.

Solutions:

1) $y = x\tan\left(\ln\left(\frac{x}{2}\right)\right)$

2) $y^2 = \frac{1}{Ae^{-x} - e^x}$

3) $y = \frac{1}{6}x^2 \ln x - \frac{5}{36}x^2 + \frac{A}{x} + B$

4) $1 + 3y^2 = (3x+2)^2$ or $y = \sqrt{3x^2 + 4x + 1}$

5) $y = x^2 + A\sqrt{1 - x^2}$

6) $y = Ax^3 - x^2 + Bx$

7) $x + y - 1 = 3e^{x-y}$