

Inequalities

- Avoid multiplying or dividing an inequality by an unknown
(because we cannot tell whether we need to reverse the inequality sign)
- Make the R H S zero.
- Write LHS as a single fraction, factorized as much as possible
- Find the 'critical values' of x
- Use a table to determine the sign of LHS

Examples

$$\textcircled{1} \quad \frac{x-3}{2(x+2)} > \frac{1}{x-4}$$

$$\frac{x-3}{2(x+2)} - \frac{1}{x-4} > 0$$

$$\frac{(x-3)(x-4) - 2(x+2)}{2(x+2)(x-4)} > 0$$

$$\frac{x^2 - 7x + 12 - 2x - 4}{2(x+2)(x-4)} > 0$$

$$\frac{(x-8)(x-1)}{2(x+2)(x-4)} > 0$$

		-2	0	1	4	8	
$x-8$	-	-	-	-	-	-	+
$x-1$	-	-	-	+	+	+	+
$x+2$	-	-	+	+	+	+	+
$x-4$	-	-	-	-	+	+	+
LHS	+	-	-	+	-	-	+

Solution: $x < -2$ or $1 < x < 4$ or $x > 8$

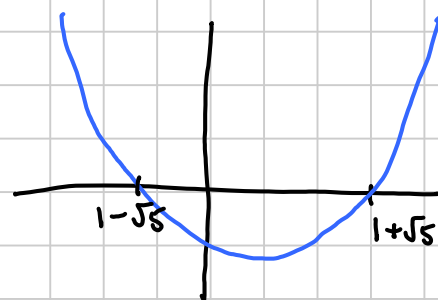
②
$$\frac{x^2 - 2x - 4}{x^2 - 2x + 2} > 0$$

Neither top nor bottom will factorize
Complete the square:

$$\frac{(x-1)^2 - 5}{(x-1)^2 + 1} > 0$$

Bottom is always +ve.

Top: $(x-1)^2 - 5 = 0$
 $\Rightarrow (x-1)^2 = 5$
 $x-1 = \pm\sqrt{5}$
 $x = 1 \pm \sqrt{5}$

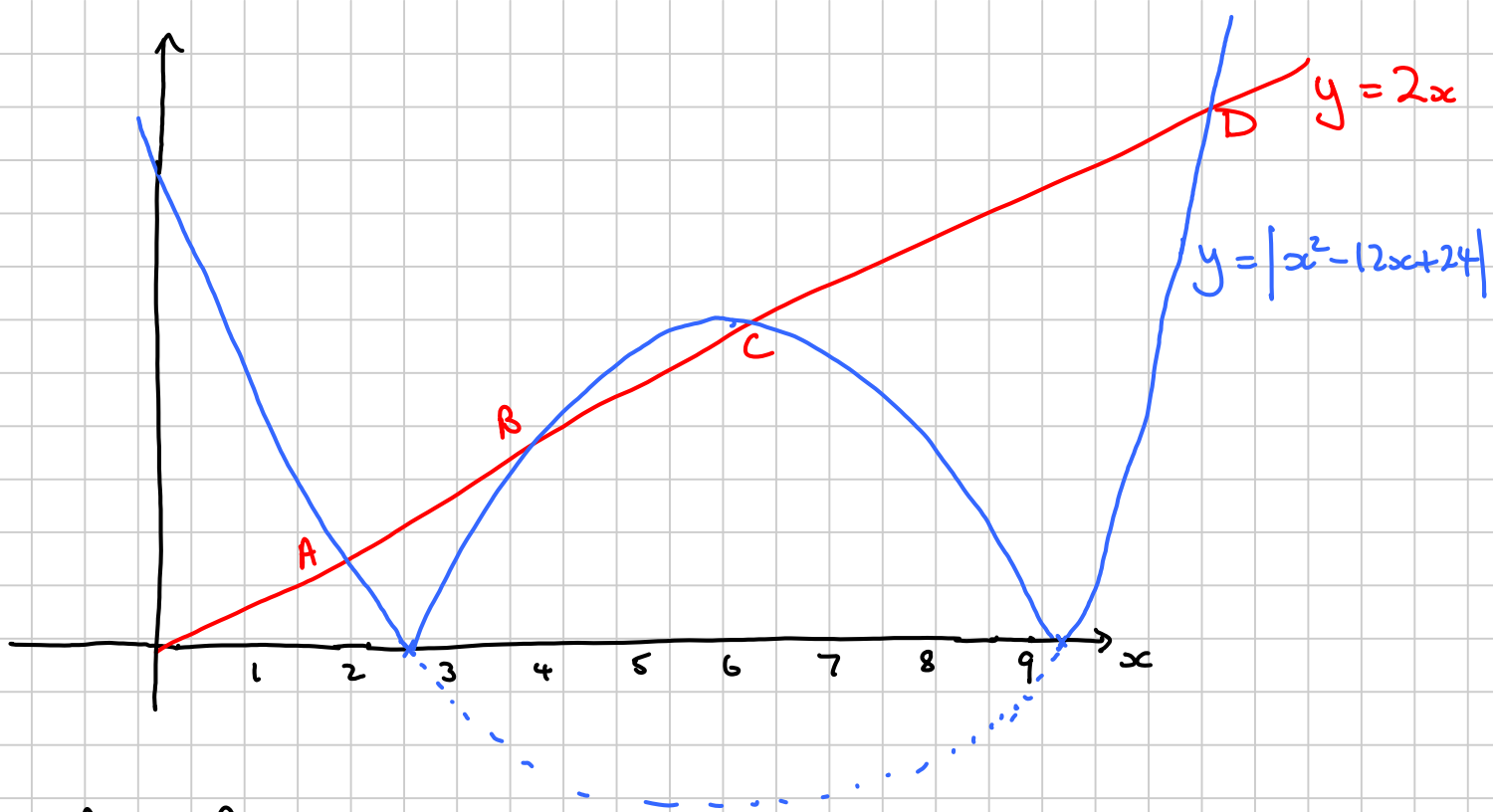


Top is +ve when $x < 1 - \sqrt{5}$ or $x > 1 + \sqrt{5}$

- For modulus inequalities always draw a graph

③ $|x^2 - 12x + 24| < 2x$

Solve $x^2 - 12x + 24 = 0$
$$x = \frac{12 \pm \sqrt{144 - 96}}{2}$$
$$= \frac{12 \pm 4\sqrt{3}}{2} = 6 \pm 2\sqrt{3}$$



At A and D,

$$x^2 - 12x + 24 = 2x$$

$$x^2 - 14x + 24 = 0$$

$$(x - 2)(x - 12) = 0$$

$$x = 2 \text{ (at A)} \quad \text{or} \quad x = 12 \text{ (at D)}$$

At B and C

$$-(x^2 - 12x + 24) = 2x$$

$$0 = x^2 - 10x + 24$$

$$0 = (x - 4)(x - 6)$$

$$x = 4 \text{ (at B)} \quad \text{or} \quad x = 6 \text{ (at C)}$$

So solution is $2 < x < 4$ or $6 < x < 12$

P 6 Ex 1.2 Q 1g, 3, 4
 P 9 Ex 1.3 Q 2e, 3b, 4f, 5f
 P 16 Ex 1.5 Q 1df, 2df, 5b, 6d