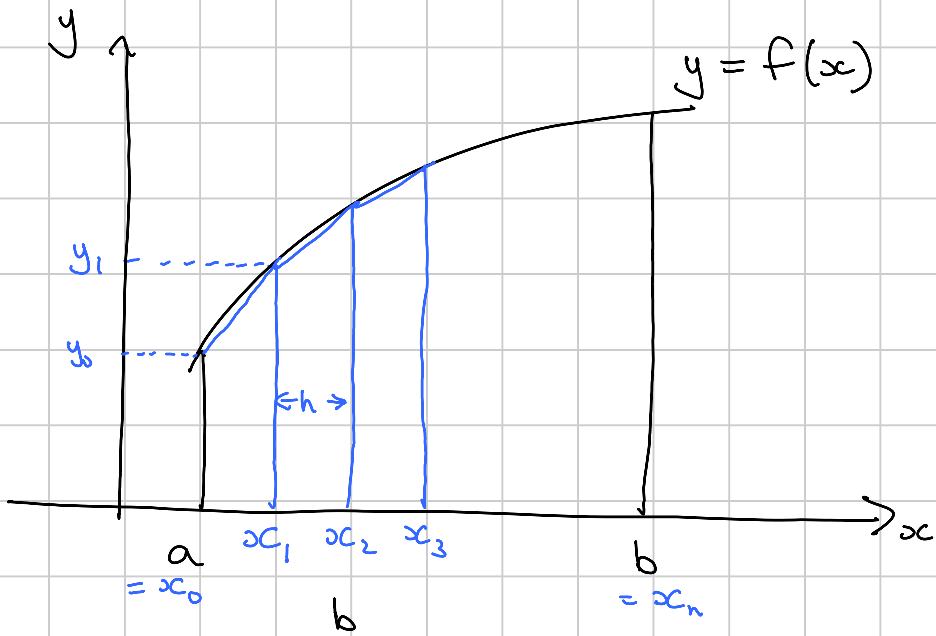
Note Title 19/04/2010

(9) The Trapezium Rule

Many integrals cannot be found exactly. So we may need to use an approximation to find the area under a curve.



To find $\int_a^b f(x)\,dx$

We divide the area into n trapezia, by drawing $n+1$ lines at $x_0, x_1 \ldots x_n$.

Each trapezium has $h = \frac{b-a}{n}$

The heights of the lines are $y_0, y_1 \ldots y_n$ where $y_i = f(x_i)$.

The area of the first trapezium $= \frac{1}{2}(y_0 + y_1)h$

" second " $= \frac{1}{2}(y_1 + y_2)h$

" third " $= \frac{1}{2}(y_2 + y_3)h$

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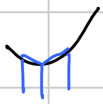
" last $= \frac{1}{2}(y_{n-1} + y_n)h$

So $$\boxed{\int_a^b f(x)\,dx \approx \tfrac{1}{2}h\left(y_0 + y_n + 2\left(y_1 + y_2 + \cdots y_{n-1}\right)\right) \quad \left(\text{where } h = \frac{b-a}{n}\right)}$$

(this is in the formula book).

Notes

(1) If the curve is  the trapezium rule will give an UNDERESTIMATE

If the curve is  the trapezium rule will give an OVERESTIMATE

(2) We can improve the accuracy of the estimate by increasing the value of n.

Example

(1) Find $\int_{-1}^{1} \sin(e^x)\,dx$ using the trapezium rule with 4 strips

x	-1	-0.5	0	0.5	1
y	0.360	0.570	0.841	0.997	0.411

$\left(h = \frac{1-(-1)}{4} = 0.5\right)$

(NB MUST USE RADIANS)

Area $\approx \frac{1}{2} \times 0.5 \times \left(0.360 + 0.411 + 2\left(0.570 + 0.841 + 0.997\right)\right)$

$\approx \underline{\underline{1.397}}$

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