The Modulus Function

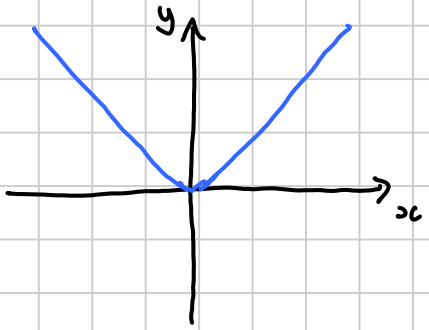
Note Title 29/11/2012

This is written as $|x|$, and is the absolute value of x ignoring any $-$ sign.

ie, if $x = 7$, $|x| = 7$
if $x = -3$, $|x| = 3$

Note that $|a||b| = |ab|$
but $|a| + |b| \neq |a + b|$ (e.g. $|-4| + |3| = 7$, $|-4+3| = 1$)

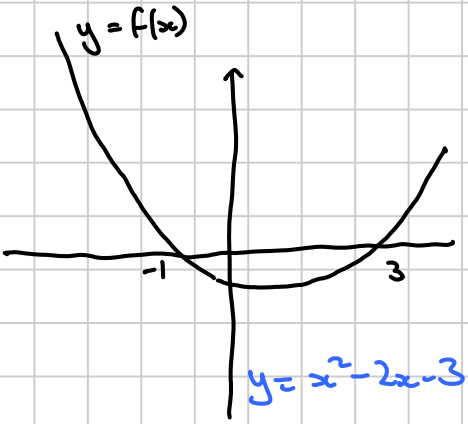
The graph of $y = |x|$ is

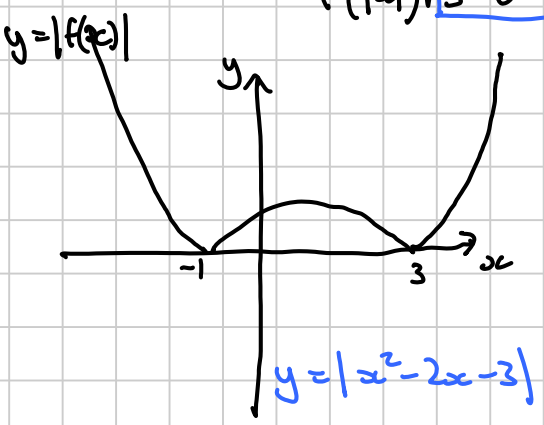


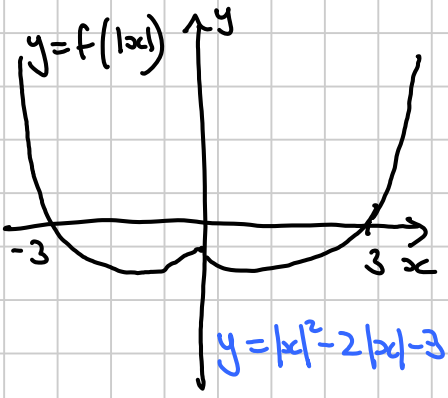
We can develop rules for the graphs of more complex modulus functions.

e.g. $f(x) = x^2 - 2x - 3$

x	-4	-3	-2	-1	0	1	2	3	4
$f(x)$	21	12	5	0	-3	-4	-3	0	5
$\|f(x)\|$	21	12	5	0	3	4	3	0	5
$f(\|x\|)$	5	0	-3	-4	-3	-4	-3	0	5





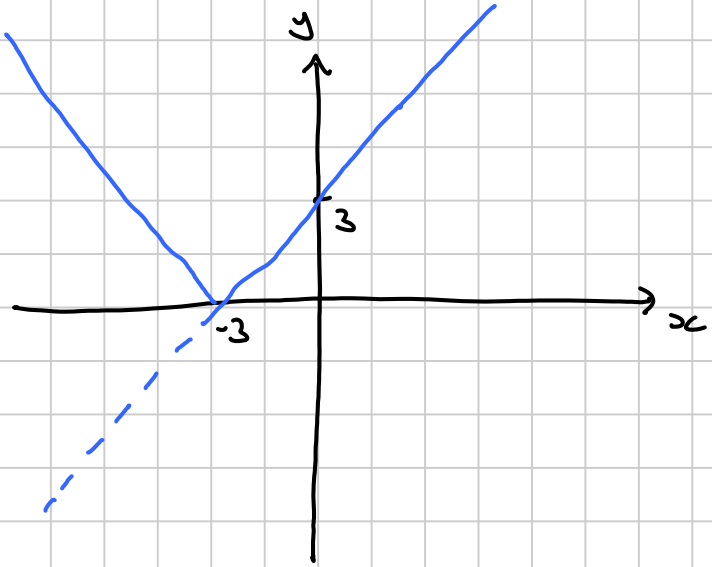


The graph of $y = |f(x)|$ is the same as the graph of $y = f(x)$, with any parts below the x-axis reflected in the x-axis.

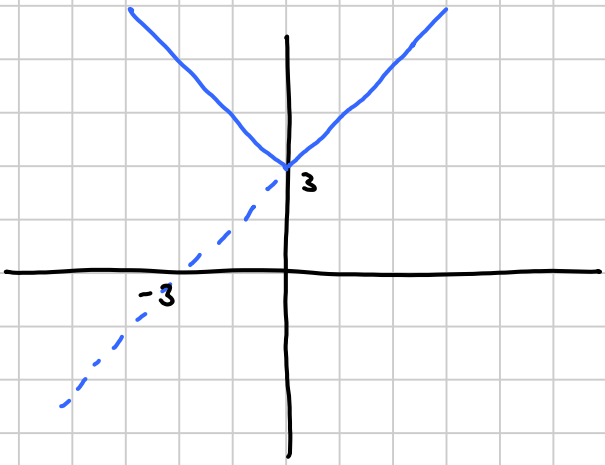
The graph of $y = f(|x|)$ is the same as the graph of $y = f(x)$, with the part to the left of the y-axis rubbed out and replaced by the reflection of the part to the right of the y-axis

More examples

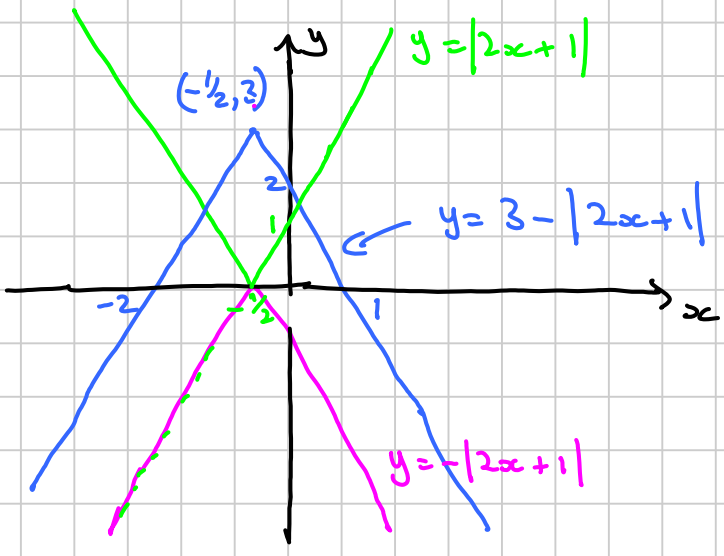
① $y = |x+3|$



② $y = |x| + 3$



④ $y = 3 - |2x+1|$
(or $y = -|2x+1| + 3$)



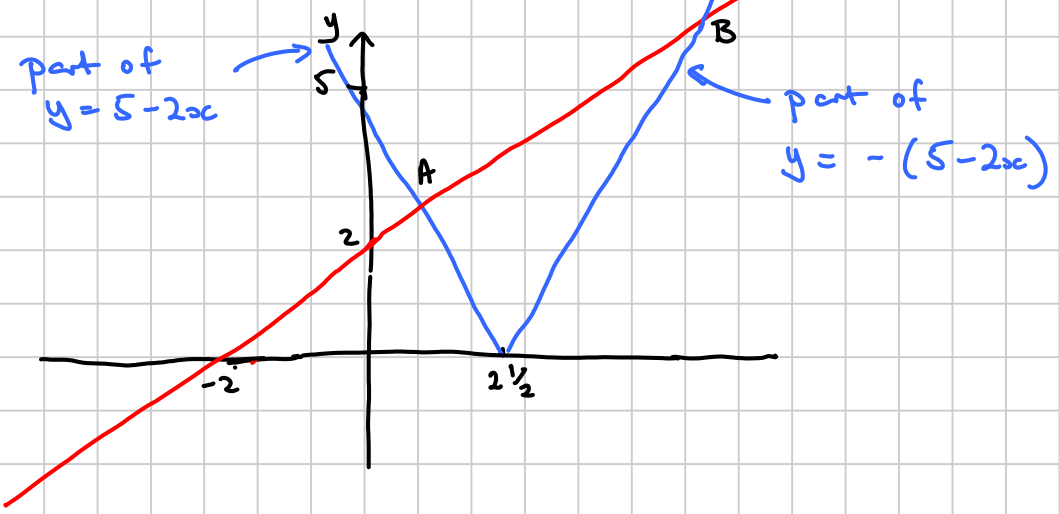
Equations and inequalities involving the modulus function

To solve these it is best to sketch a graph first.

Examples

① Solve (a) $|5-2x| = x+2$

(b) $|5-2x| < x+2$



(a) At A, $5-2x = x+2$

$3 = 3x$

$x = 1$

At B $-(5-2x) = x+2$

$-5+2x = x+2$

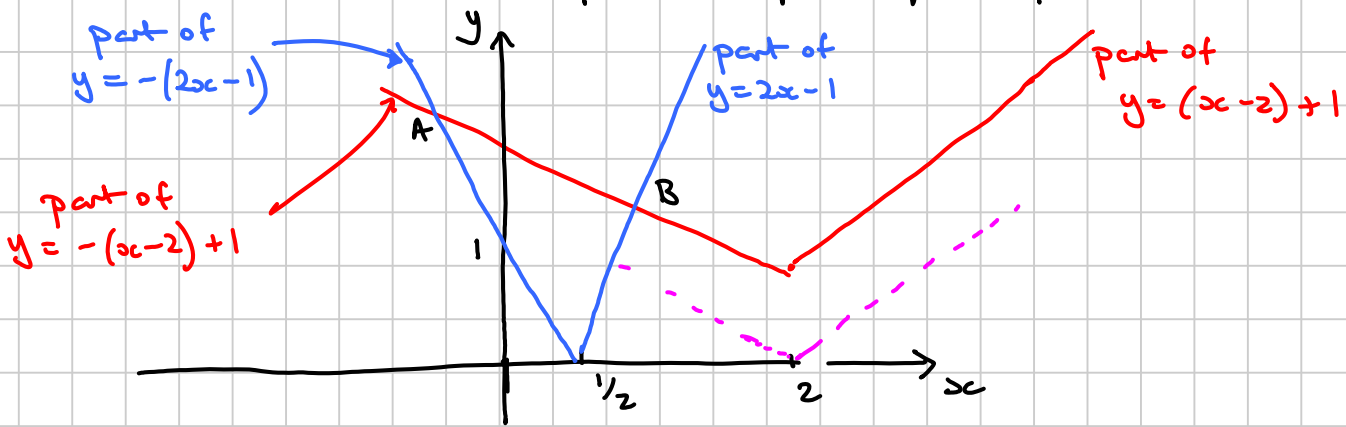
$x = 7$

(b) Graph of $y = |5-2x|$ is below graph of $y = x+2$ for

$1 < x < 7$

② Solve $|2x-1| - |x-2| = 1$

Write as $|2x-1| = |x-2| + 1$



At A, $-(2x-1) = -(x-2)+1$

$-2x+1 = 3-x$

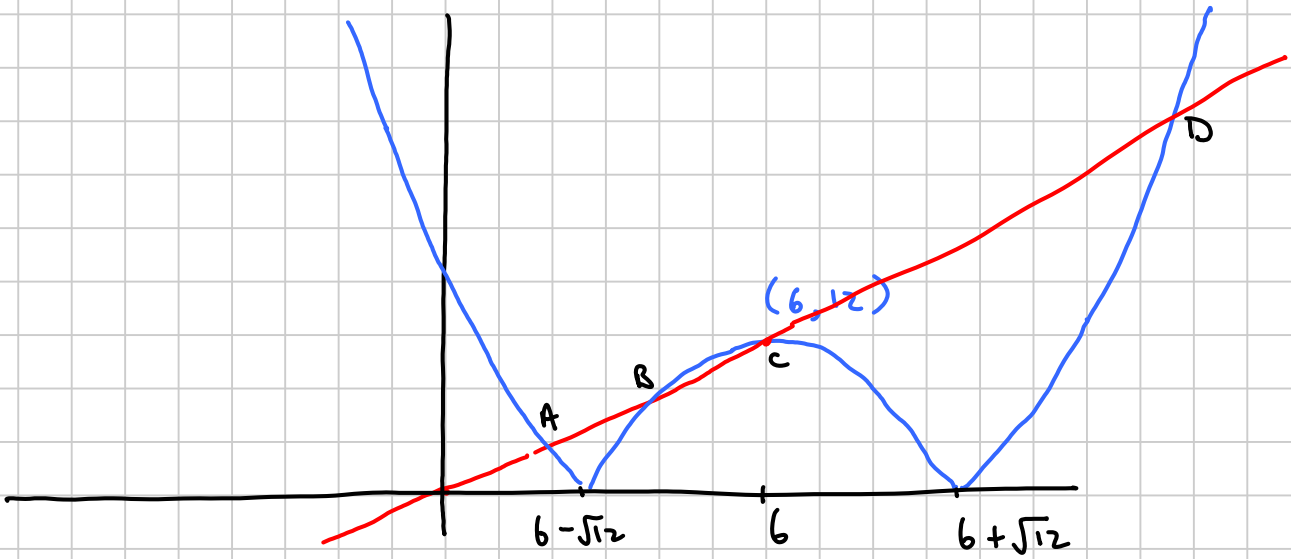
$-2 = x$

At B, $2x-1 = 3-x$

$3x = 4$

$x = 4/3$

③ Solve $|x^2-12x+24| > 2x$



LHS $y = x^2-12x+24$

$= (x-6)^2-12$

If $y=0$, $(x-6)^2-12=0$

$(x-6)^2 = 12$

$x = 6 \pm \sqrt{12}$

At A & D, $x^2-12x+24 = 2x$

$x^2-14x+24 = 0$

$(x-2)(x-12) = 0$

$x=2$ (at A) or $x=12$ (at D)

At B & C $-(x^2-12x+24) = 2x$

$-x^2+12x-24 = 2x$

$0 = x^2-10x+24$

$0 = (x-4)(x-6)$

$x=4$ (at B) or $x=6$ (at C)

So $|x^2-12x+24| > 2x$ if $x<2$ or $4<x<6$ or $x>12$

Ex 5A Q 1 (abdefghi), 2(ab)
5B Q 1, 2, 3, 4
5C Q 1, 2, 3, 5

p 67 Ex 5D Q 1
Ex 5E Q 1, 2, 3