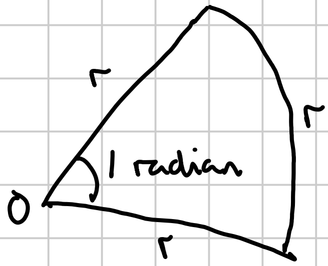
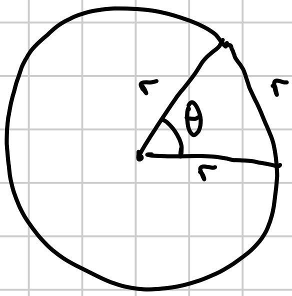
Note Title 09/01/2012

TRIGONOMETRY

Radians These are an alternative to degrees for measuring angles. If we have an arc of a circle equal to the radius, the angle it subtends is one radian



The symbol for 1 radian is 1^c
(If an angle has no symbol, it is assumed to be radians.)



Whole circumference $= 2\pi r$

$$\theta = \frac{\cancel{r}}{2\pi \cancel{r}} \times 360^\circ$$

$$= 57.3^\circ \quad (\text{approx})$$

So $\boxed{1^c \approx 57.3^\circ}$

$$1^c = \frac{360}{2\pi}^\circ$$

$$\boxed{2\pi^c = 360^\circ = \text{one whole turn}}$$

$$\boxed{\pi^c = 180^\circ}$$

Common angles

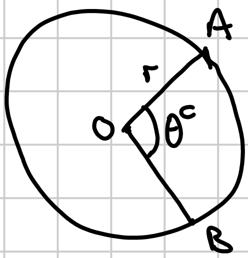
Degrees	30°	60°	45°	90°	120°	135°	150°
Radians	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$

To convert from degrees to radians, multiply by $\frac{\pi}{180}$

To convert from radians to degrees, multiply by $\frac{180}{\pi}$

p 81 Ex 6A Q 1 acegi, 2 ace, 3 ace
4 bghjkmn, 5 a-f.

Arc length and sector area



length of arc AB $= \dfrac{\theta}{\cancel{2\pi}} \times \cancel{2\pi} r$

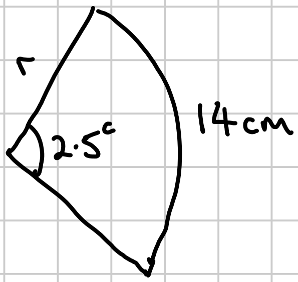
$= r\theta$

Area of sector OAB $= \dfrac{\theta}{2\cancel{\pi}} \times \cancel{\pi} r^2$

$= \frac{1}{2} r^2 \theta$

Examples

1



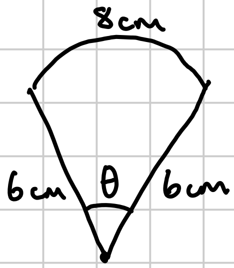
Find r

$2.5r = 14$

$r = \dfrac{14}{2.5}$

$= \underline{\underline{5.6\text{cm}}}$

2



Find the area of this shape.

$6\theta = 8$

$\theta = 8/6 = \frac{4}{3}^c$

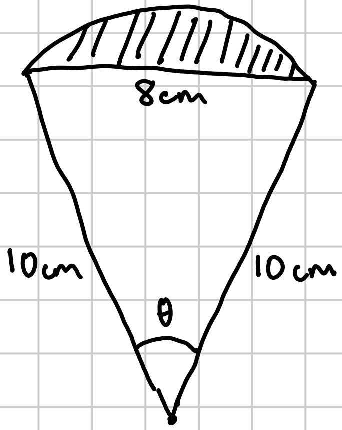
Area $= \frac{1}{2} \times 6^2 \times \frac{4}{3}$

$= \underline{\underline{24\text{cm}^2}}$

p 83 Ex 6B Q 1 a(i) b(i) c(i), 4, 5

p 87 Ex 6C Q 1 abdef, 2 ac, 3, 4, 5, 6, 9, 12

③



Find the area of the shaded segment.

$$\cos\theta = \frac{10^2 + 10^2 - 8^2}{2 \times 10 \times 10}$$

$$= \frac{136}{200}$$

$$\theta = \cos^{-1}\left(\frac{136}{200}\right)$$

$$= 0.823^c$$

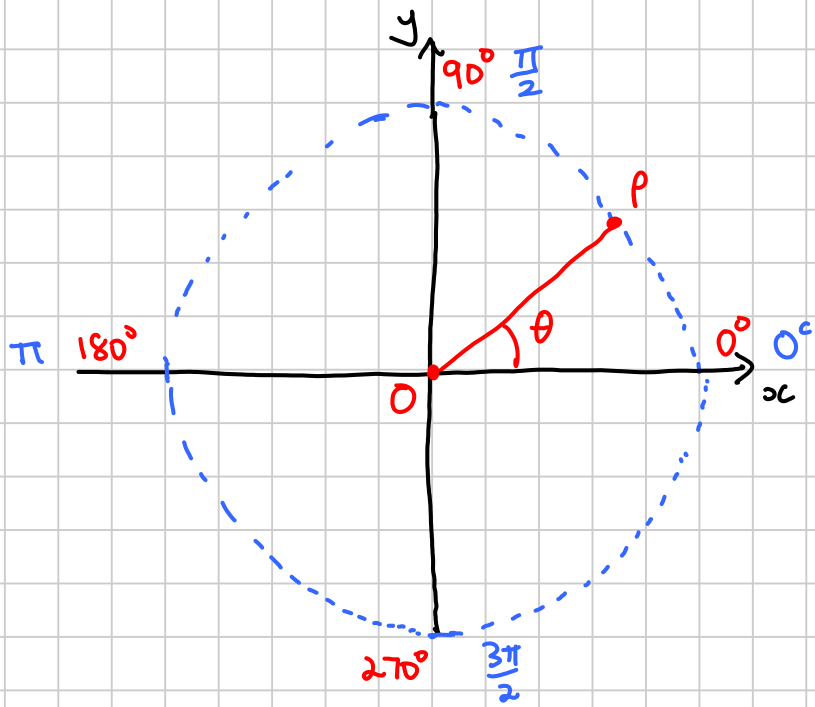
Area of sector $= \frac{1}{2} \times 10^2 \times 0.823 = 41.15 \text{ cm}^2$

Area of triangle $= \frac{1}{2} \times 10 \times 10 \times \sin 0.823 = 36.66 \text{ cm}^2$

Area of segment $= 41.15 - 36.66 = \underline{\underline{4.49}} \text{ cm}^2$

Sine, Cosine, Tangent of angles of any size

Consider a line OP, 1 unit long which rotates around the origin O, starting at the positive x-axis, and rotating anticlockwise (+ve angles) or clockwise (−ve angles).



Then,

$\cos\theta$ = x-coord of P
$\sin\theta$ = y-coord of P
$\tan\theta$ = gradient of OP

So

$$-1 \leq \cos\theta \leq 1 \qquad (\cos 180° = -1, \quad \cos 0° = 1)$$
$$-1 \leq \sin\theta \leq 1 \qquad (\sin 270° = -1 \quad \sin 90° = 1)$$

$\tan\theta$ can be of any size.

(Quadrant diagram:

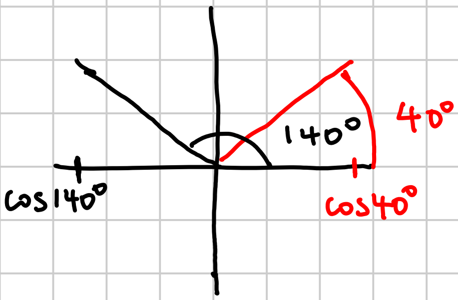
2nd Q	1st Quadrant
S: $\boxed{\sin\ +ve}$, $\cos\ -ve$, $\tan\ -ve$	A: $\boxed{\sin\ +ve,\ \cos\ +ve,\ \tan\ +ve}$
T: $\sin\ -ve$, $\cos\ -ve$, $\boxed{\tan\ +ve}$	C: $\sin\ -ve$, $\boxed{\cos\ +ve}$, $\tan\ -ve$
3rd Q	4th Q

Sisters
Always
Tease
Cats

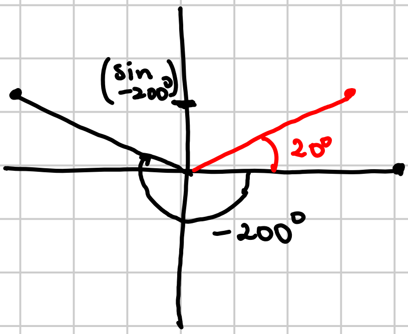
Examples

① Write as sin, cos or tan of an acute angle.

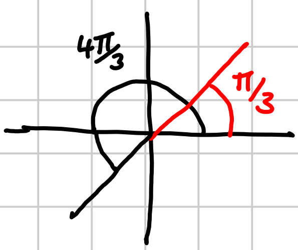
(a) $\cos 140^\circ \quad = -\cos 40^\circ$



(b) $\sin(-200^\circ) \quad = \sin 20^\circ$

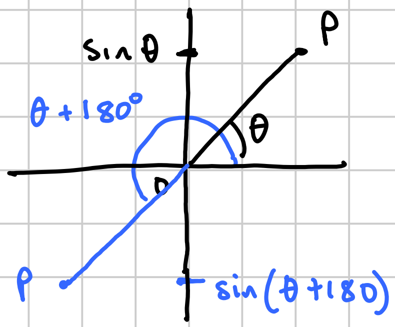


(c) $\tan \frac{4\pi}{3} \quad (= 1\tfrac{1}{3}\pi) \quad = \tan \frac{\pi}{3}$



p117 Ex 8C
Q 1 a-e, p-t

② Express $\sin(\theta + 180)$ in terms of $\sin\theta$ (assume θ is acute).



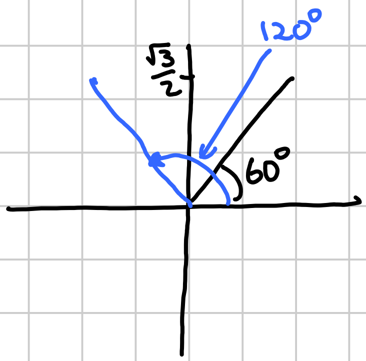
$$\sin(\theta + 180) = -\sin\theta$$

③ Solve these equations in the interval indicated:

(a) $\sin\theta = \frac{\sqrt{3}}{2}$ $\quad (0 < \theta < 720°)$

$$\theta = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$$
$$= 60°$$

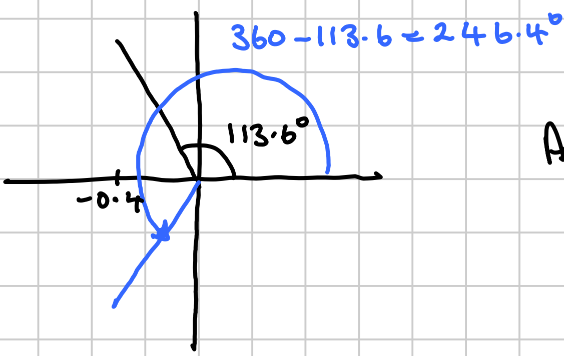
(this is called the Principal Value (PV) of θ)



All solutions : $\theta = 60°, 120°, 420°, 480°$

(b) $\cos\theta = -0.4$ $\quad (-360° \leq \theta \leq 360°)$

PV $\quad \theta = \cos^{-1}(-0.4)$
$\quad = 113.6°$



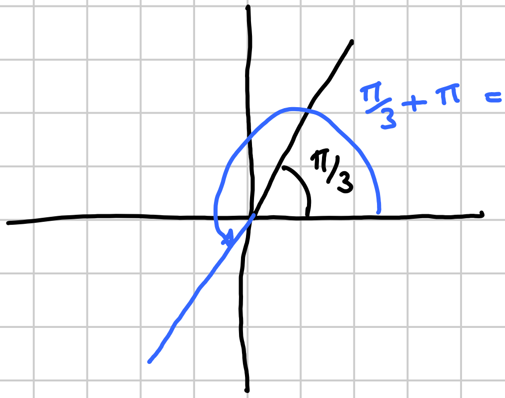
All solns : $\theta = 113.6°, 246.4°, -113.6°, -246.4°$

(c) $\tan\theta = \sqrt{3}$ $\quad (0 < \theta < 4\pi)$

calculator into radian mode!

PV $\theta = \tan^{-1}(\sqrt{3})$

$= \frac{\pi}{3}$



$\frac{\pi}{3} + \pi = \frac{4\pi}{3}$

All solns: $\theta = \underline{\underline{\frac{\pi}{3}}}$

$\underline{\underline{\frac{4\pi}{3}}}$

$\frac{\pi}{3} + 2\pi = \underline{\underline{\frac{7\pi}{3}}}$

$\frac{4\pi}{3} + 2\pi = \underline{\underline{\frac{10\pi}{3}}}$

p 117 Ex 8C Q 2a, c, 3 a c g i ← Like Example 2

p 148 Ex 10B Q 1a–e, 2, <u>3abce</u> ← Like Ex 3

④ Solve these equations in the interval given :–

(a) $\sin 3\theta = -\frac{1}{2}$ $\quad (0 \leq \theta \leq 180°)$

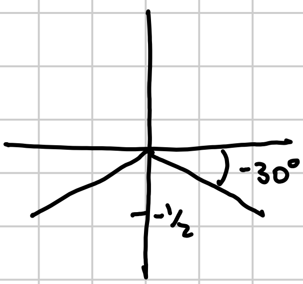
Let $\alpha = 3\theta$ $\quad \Rightarrow \quad (0 \leq \alpha \leq 540°)$

$\sin\alpha = -\frac{1}{2}$

PV $\alpha = \sin^{-1}(-\frac{1}{2})$

$= -30°$

(NB PV is <u>not</u> one of allowed solutions)



All solns $\alpha = 210°, 330°,$ $\quad$ (570° is too big...)

$\Rightarrow 3\theta = 210°, 330°$

$\theta = \underline{\underline{70°, 110°}}$

(b) $2\cos\left(2\theta + \frac{\pi}{3}\right) = 0.8 \qquad (-\pi < \theta < \pi)$

CAN divide eqn by this 2

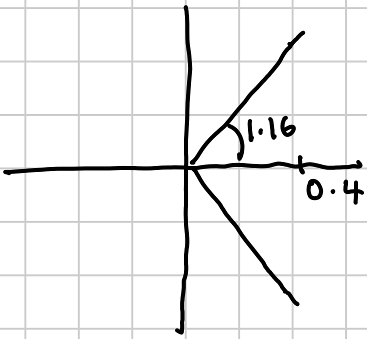
CAN'T divide eqn by this 2 (it's inside the cos function)

$(\div 2) \qquad \cos\left(2\theta + \frac{\pi}{3}\right) = 0.4$

Let $\alpha = 2\theta + \frac{\pi}{3} \qquad \Rightarrow \qquad \left(-\frac{5\pi}{3} < \alpha < \frac{7\pi}{3}\right)$

$-5.24 < \alpha < 7.33$

$$\cos\alpha = 0.4$$
$$\alpha = \cos^{-1}(0.4)$$
$$= 1.16^c.$$



All solns $\alpha = 1.16,$
or $2\pi - 1.16 = 5.12$
or $2\pi + 1.16 = 7.44$ (too big)
or -1.16
or -5.12

$$2\theta + \frac{\pi}{3} = 1.16,\ 5.12,\ -1.16 \text{ or } -5.12$$

$$\theta = 0.056,\ 2.04,\ -1.10, \text{ or } -3.08$$

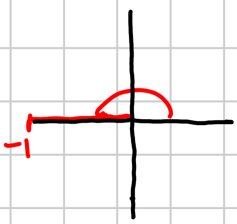
5 Solve $2\cos^2\theta + \cos\theta - 1 = 0 \qquad (0 \le \theta \le 360°)$

Let $c = \cos\theta$

$$2c^2 + c - 1 = 0$$
$$(c+1)(2c-1) = 0$$
$$c = -1 \quad \text{or} \quad c = \frac{1}{2}$$

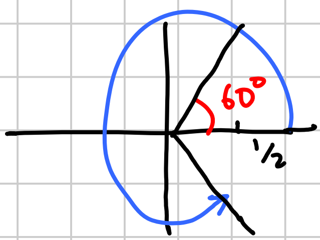
$\cos\theta = -1$

$\theta = 180°$



$\cos\theta = \frac{1}{2}$

PV $\theta = \cos^{-1}\left(\frac{1}{2}\right)$
$= 60°$



or $\theta = 300°$

All soln's $180°, 60°, 300°$

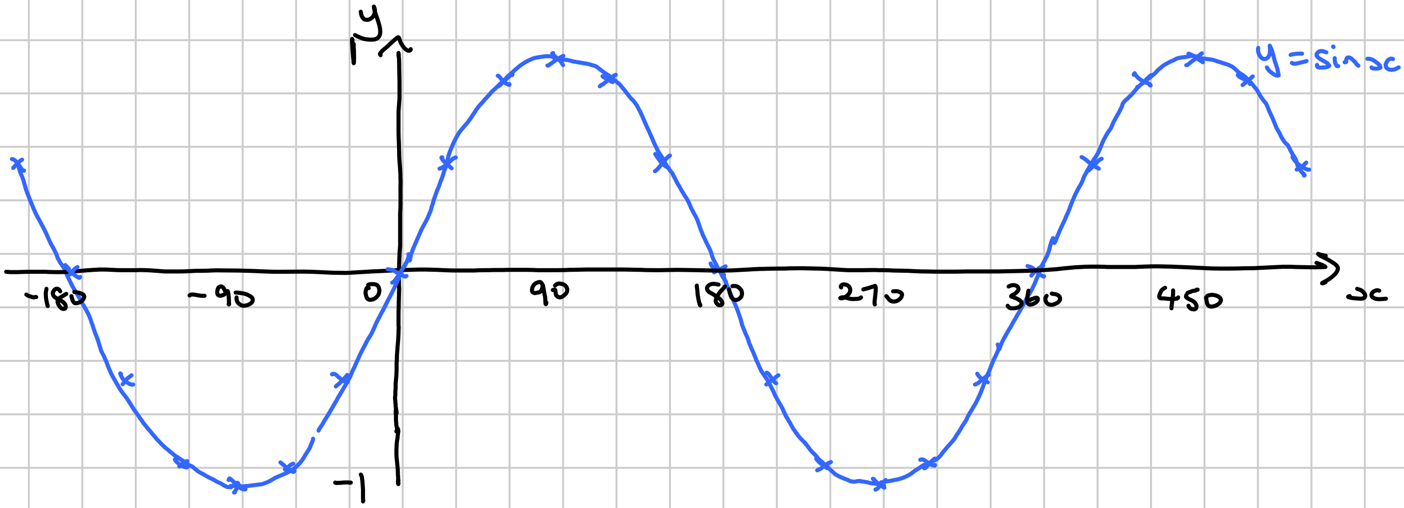
Ex 10C Q 1dej, 3abc

Graphs of Trig Functions

$y = \sin x$

x	0	30	60	90	120	150	180	210	240	270	300	330	360
y	0	0.5	0.87	1	0.87	0.5	0	-0.5	-0.87	-1	-0.87	-0.5	0

Since $\sin(x + 360) = \sin x$, the graph is PERIODIC, with a period of 360° (or 2π) ie, it repeats every 360°.



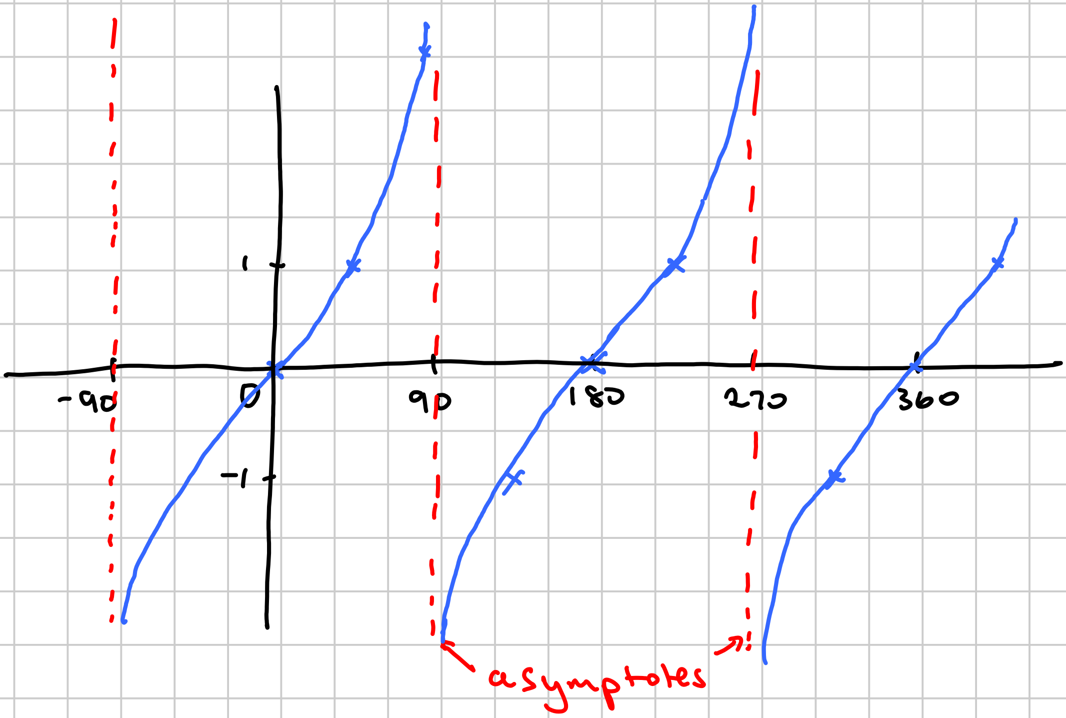
$y = \cos x$

x	0	30	60	90
y	1	0.87	0.5	0

The graph of $\cos x$ is the same as the graph of $\sin x$, but translated 90° to the left.

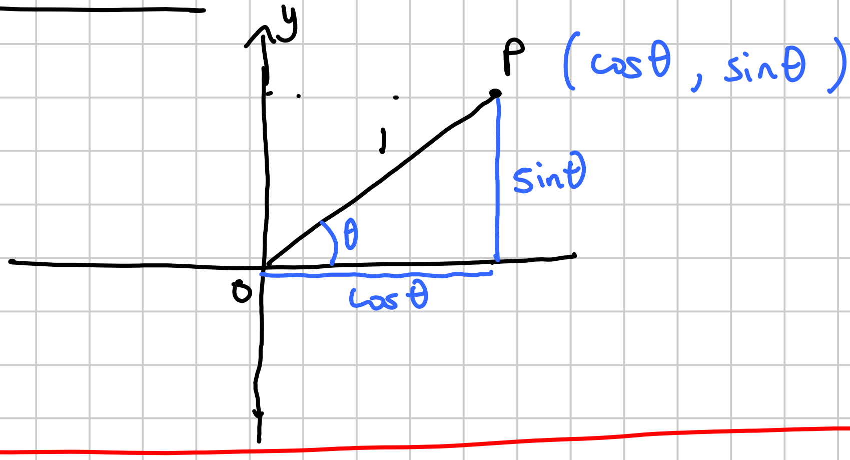
$y = \tan x$

x	0	45	90	135	180	225	270	315	360
y	0	1	∞	-1	0	1	∞	-1	0



The graph is periodic with period 180° (between asymptotes 180° apart.

Trig Identities



By Pythagoras, $\cos^2\theta + \sin^2\theta = 1$

Gradient : $\dfrac{\sin\theta}{\cos\theta} = \tan\theta$

We can use these identities to prove other identities or to solve equations.

① Prove the identity $\tan\theta + \dfrac{1}{\tan\theta} = \dfrac{1}{\sin\theta\cos\theta}$

NB When proving an identity, don't start by writing down the whole thing. Start with one side and work on it to make it equal the other side.

$$\text{LHS} = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}$$

$$\frac{2}{3} + \frac{3}{5} = \frac{10+9}{15}$$

$$= \frac{1}{\cos\theta\sin\theta}$$

$$= \text{RHS}$$

QED

② Solve $2\sin^2\theta = 1 + \cos\theta$ for $0° \leq x \leq 360°$

Since $\cos^2\theta + \sin^2\theta = 1$, $\sin^2\theta = 1 - \cos^2\theta$.

So $2(1 - \cos^2\theta) = 1 + \cos\theta$

$2 - 2\cos^2\theta = 1 + \cos\theta$

$0 = 2\cos^2\theta + \cos\theta - 1$

(and continue as in example 5 above)

③ Solve $2\sin\theta = \tan\theta$ $(0 \leq \theta \leq 2\pi)$

$2\sin\theta = \dfrac{\sin\theta}{\cos\theta}$

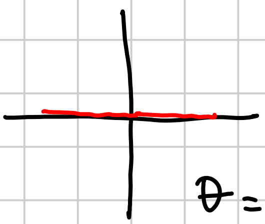
$2\sin\theta\cos\theta = \sin\theta$

$2\sin\theta\cos\theta - \sin\theta = 0$

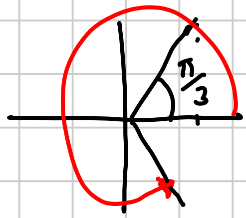
$\sin\theta(2\cos\theta - 1) = 0$

$\sin\theta = 0$ or $2\cos\theta - 1 = 0$

$\cos\theta = \frac{1}{2}$



$\theta = 0$ or π or 2π



$\theta = \frac{\pi}{3}$ or $\frac{5\pi}{3}$

Ex 10 A Q 5abe

Ex 10 D Q 1adeio, 2de, 3cde