

INTEGRATION

The reverse process to differentiation is called integration. The equivalent rules are:—

$$\text{If } \frac{dy}{dx} = x^n, \quad y = \frac{1}{n+1} x^{n+1} + c$$

(where c can be any constant)
(provided $n \neq -1$!)

$$\text{If } \frac{dy}{dx} = k, \quad y = kx + c$$

Notation The following all mean the same:—

$$\text{If } \frac{dy}{dx} = 3x^2, \quad y = x^3 + c$$

$$\text{If } f(x) = 3x^2, \quad F(x) = x^3 + c$$

$$\int 3x^2 dx = x^3 + c$$

These are like brackets round the function to integrate
Read as "the integral of $3x^2$ with respect to x ".

Examples

① Find $\int \sqrt{x} + \frac{3}{x^2} dx$

$$= \int x^{1/2} + 3x^{-2} dx$$
$$= \frac{1}{1/2} x^{1/2} + 3 \times \frac{1}{-1} x^{-1} + c$$
$$= \underline{\underline{\frac{2}{3} x^{3/2} - \frac{3}{x} + c}}$$

② If $f(x) = \frac{1}{3\sqrt{x}}$, find $F(x)$

$$f(x) = \frac{1}{3} x^{-1/2}$$

Thus 3 stays on the bottom.

$$\begin{aligned} \text{So } F(x) &= \frac{1}{3} \times \frac{1}{1/2} x^{1/2} + C \\ &= \frac{1}{3} \times 2 x^{1/2} + C \\ &= \underline{\underline{\frac{2}{3} x^{1/2} + C}} \end{aligned}$$

③ A curve has gradient $\frac{dy}{dx} = x^2 - 4x + 3$ and passes through the point $(3, 7)$. Find the equation of the curve. [Because we are given a point on the curve, we can find 'c'.]

$$\begin{aligned} y &= \frac{1}{3} x^3 - 4 \times \frac{1}{2} x^2 + 3x + C \\ y &= \frac{1}{3} x^3 - 2x^2 + 3x + C \end{aligned}$$

$$\begin{aligned} \text{When } x=3, y=7 : \quad 7 &= \frac{1}{3} \times 27 - 2 \times 9 + 9 + C \\ 7 &= C \end{aligned}$$

$$\text{Equation of curve is } y = \frac{1}{3} x^3 - 2x^2 + 3x + 7$$

④ A curve has gradient given by $\frac{dy}{dx} = \frac{\sqrt{x} + 3}{x^2}$, and passes through the point $(4, 1)$. Find the equation of the curve.

$$\begin{aligned} \frac{dy}{dx} &= \frac{x^{1/2}}{x^2} + \frac{3}{x^2} \\ &= x^{-3/2} + 3x^{-2} \end{aligned}$$

$$y = -2x^{-1/2} - 3x^{-1} + C$$

$$y = -\frac{2}{\sqrt{x}} - \frac{3}{x} + C$$

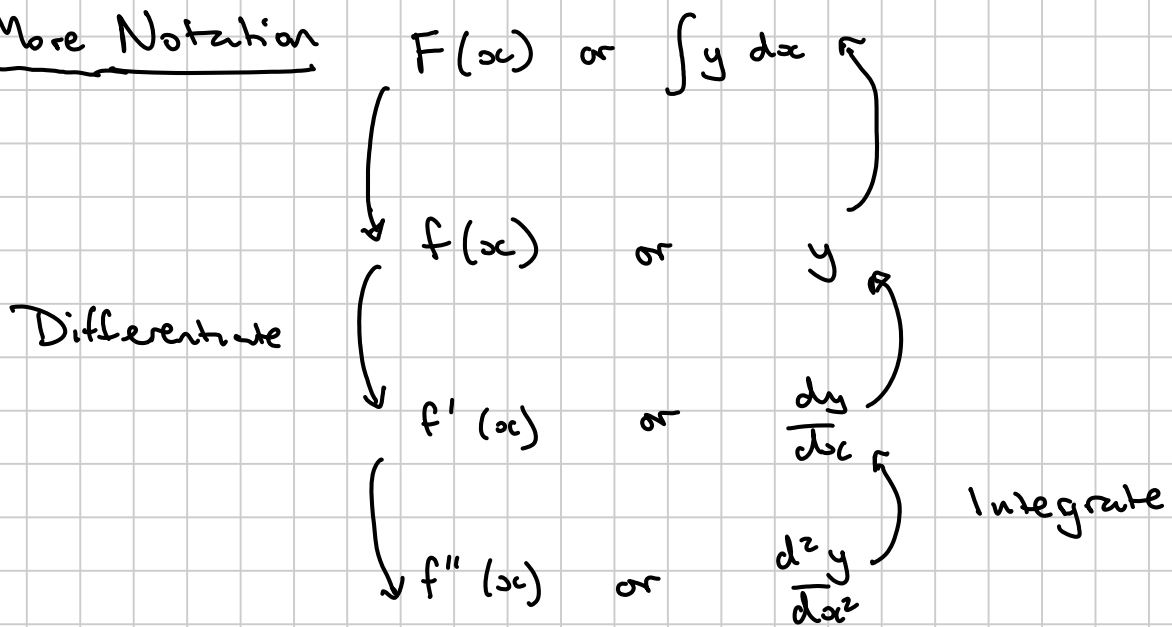
$$\text{When } x=4, y=1$$

$$1 = -\frac{2}{2} - \frac{3}{4} + C$$

$$C = 2\frac{3}{4}$$

$$\text{Equation of the curve is } \underline{\underline{y = -\frac{2}{\sqrt{x}} - \frac{3}{x} + \frac{11}{4}}}$$

More Notation



Examples

① If $f'(x) = x^2 + 4x$, find $f(x)$

$$\underline{f(x) = \frac{1}{3}x^3 + 2x^2 + C}$$

② If $\frac{d^2y}{dx^2} = 6x + 7$, find $\frac{dy}{dx}$

$$\underline{\frac{dy}{dx} = 3x^2 + 7x + C}$$

③ Find

$$\int \sqrt{x} + \frac{3}{x^2} dx$$

"The integral of"

$$= \int x^{1/2} + 3x^{-2} dx$$

"with respect to x"

$$= \frac{2}{3}x^{3/2} + 3 \times \frac{1}{-1}x^{-1} + C$$

$$= \underline{\underline{\frac{2}{3}x^{3/2} - \frac{3}{x} + C}}$$

P	123	Ex	8A	Q	1-3
P	124	Ex	8B	Q	1aceg
P	125	Ex	8C	Q	1-4
P	127	Ex	8D	Q	1, 2 bce, 3 hj
P	128	Ex	8E	Q	1ab, 3