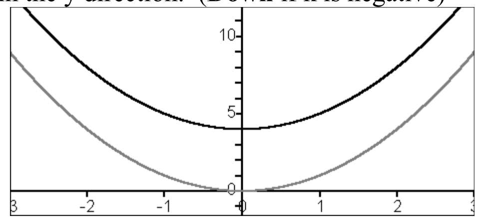
Transformations of Graphs

1)

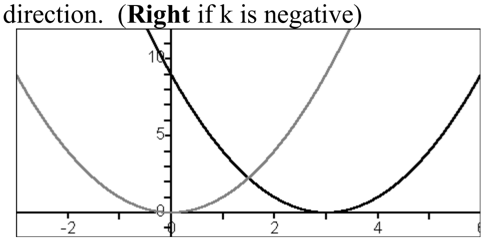
(a) The graph of $y = f(x) + k$ is the same as the graph of $y = f(x)$, but shifted **up** by k units in the y direction. (**Down** if k is negative)

Example: The graph on the right shows $y = x^2$ (in grey) and $y = x^2 + 4$ (in black)



(b) The graph of $f(x+k)$ is the same as the graph of $f(x)$, but shifted **left** by k units in the x direction. (**Right** if k is negative)

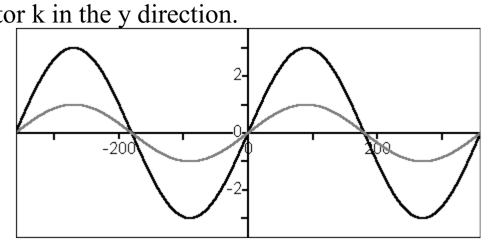
Example: The graph on the right shows $y = x^2$ (in grey) and $y = (x-3)^2$ (in black)



2)

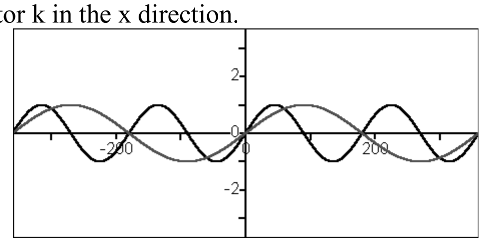
(a) The graph of $y = kf(x)$ is the same as the graph of $y = f(x)$, but stretched by a scale factor k in the y direction.
(If k is a fraction, the graph is squashed in the y-direction.)

Example: The graph on the right shows $y = \sin x$ (in grey) and $y = 3\sin x$ (in black)
(Note that when the graph is stretched, the x-coordinate of each point stays the same, but the y-coordinate is multiplied by 3)



(b) The graph of $y = f(kx)$ is the same as the graph of $y = f(x)$, but squashed by a scale factor k in the x direction.
(If k is a fraction, the graph is stretched in the x-direction.)

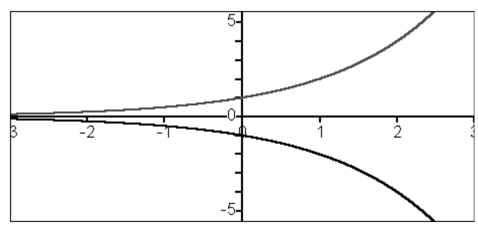
Example: The graph on the right shows $y = \sin x$ (in grey) and $y = \sin(2x)$ (in black)
(Note that when the graph is squashed, the y-coordinate of each point stays the same, but the x-coordinate is divided by 2)



3)

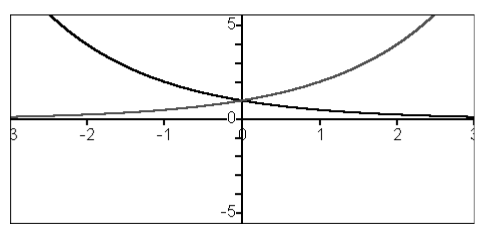
(a) The graph of $y = -f(x)$ is the same as the graph of $y = f(x)$, but reflected in the x-axis.

Example: The graph on the right shows $y = 2^x$ (in grey) and $y = -2^x$ (in black)



(b) The graph of $y = f(-x)$ is the same as the graph of $y = f(x)$, but reflected in the y-axis.

Example: The graph on the right shows $y = 2^x$ (in grey) and $y = 2^{-x}$ (in black)



4) The graph of $y = f^{-1}(x)$ is the same as the graph of $y = f(x)$, but reflected in the line $y=x$.

Example: The graph on the right shows $y = x^2$ (for $x \geq 0$) (in grey) and $y = \sqrt{x}$ (in black)