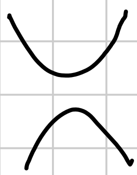
CURVE SKETCHING

When we SKETCH a graph of $y = f(x)$, we are not plotting lots of points – we are interested in the general shape of the graph:

- What happens as $x \to \infty$ and $x \to -\infty$
- Where it crosses the axes
- Turning points
- Asymptotes – lines which the graph gets close to but never touches.

Quadratic Graphs

$y = ax^2 + bx + c$ looks like:



if $a > 0$
if $a < 0$
} a parabola

c is the y-intercept.

Examples

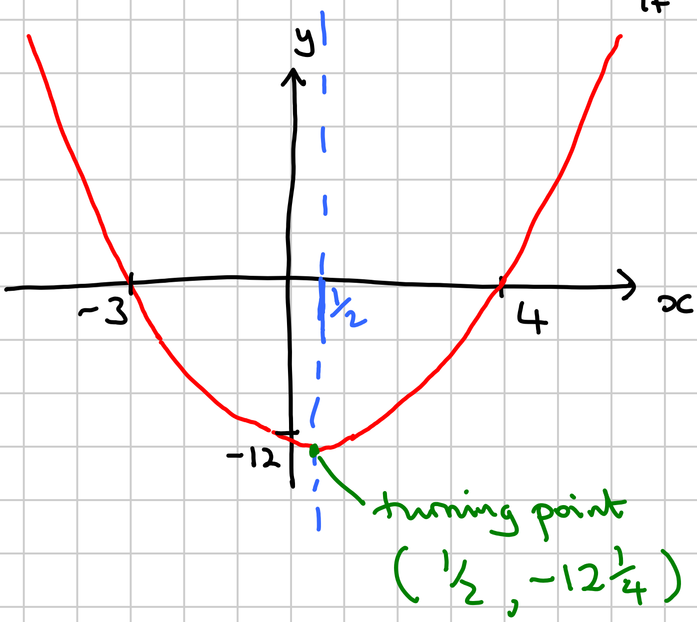
① $y = x^2 - x - 12$

If $x = 0$, $y = -12$

If $y = 0$, $x^2 - x - 12 = 0$

$(x-4)(x+3) = 0$

$x = 4$ or $x = -3$



Line of symmetry half way between 4 and −3. ie, at $x = \frac{1}{2}$

At TP, $y = \left(\frac{1}{2}\right)^2 - \frac{1}{2} - 12$

$= -12\frac{1}{4}$

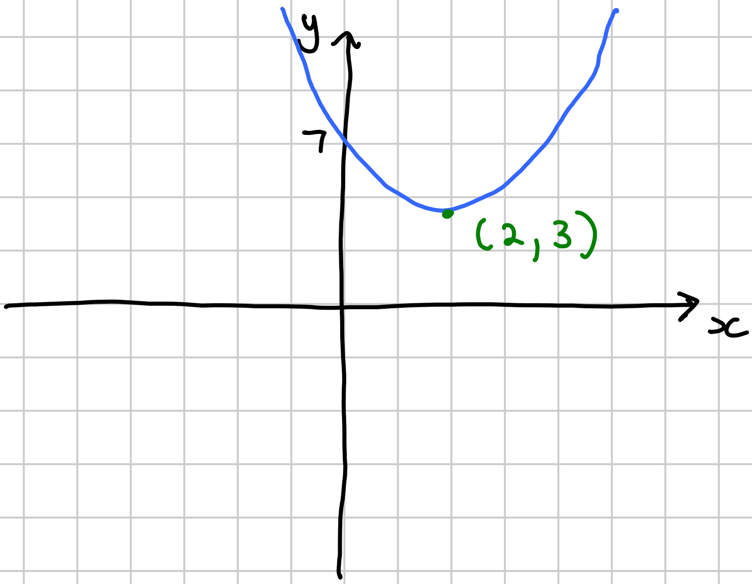
(2) $y = x^2 - 4x + 7$ If $x = 0$, $y = 7$
If $y = 0$, $x^2 - 4x + 7 = 0$

But $b^2 - 4ac = 16 - 28 = -12$ so this equation has no real roots. So the graph doesn't cross the x-axis.

Complete the square : $y = x^2 - 4x + 4 + 3$
$= (x-2)^2 + 3$

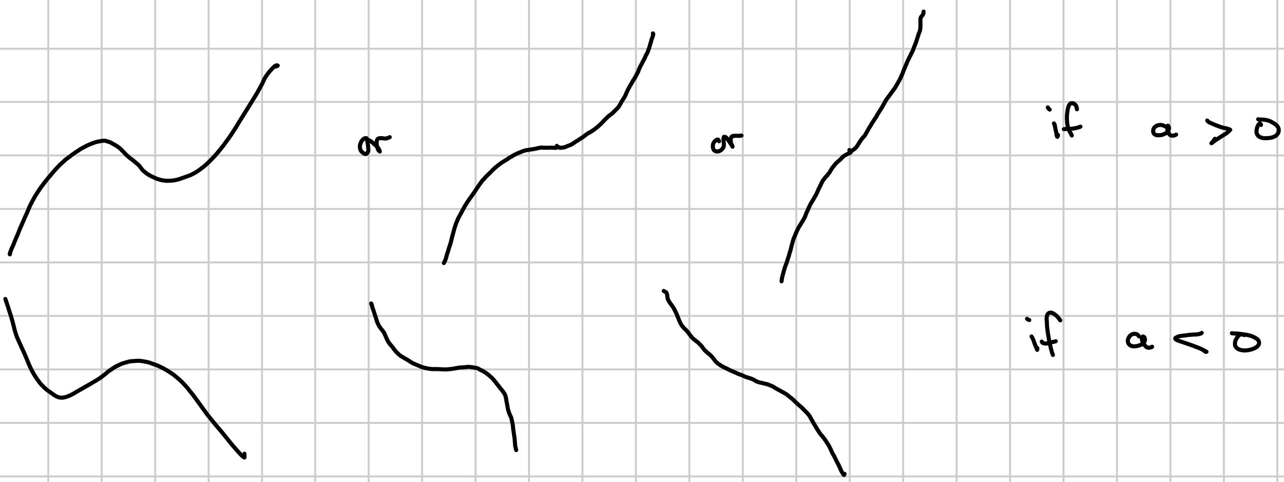
Minimum value of y is 3 and this occurs when $x = 2$

[Because for any other value of x, $(x-2)^2$ is POSITIVE.]



Cubic Graphs

$y = ax^3 + bx^2 + cx + d$ looks like:—

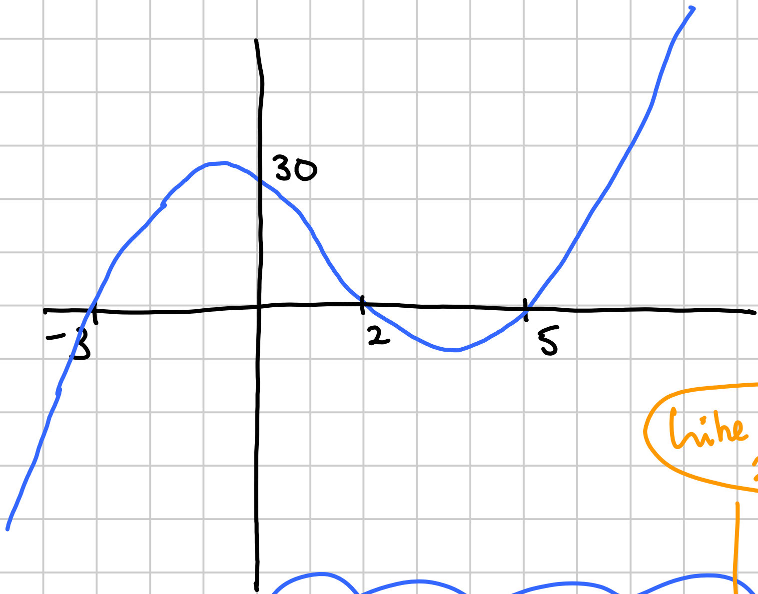


Examples

① $y = (x-2)(x-5)(x+3)$

If $x = 0$, $y = -2 \times -5 \times 3$
$= 30$

If $y = 0$, $x = 2$ or 5 or -3



If we were to multiply out the brackets we would get $+x^3$ so curve is

Like example 2

Like example 1

P 22 Ex 2F Q 1 b c i j

P 42 Ex 4A Q 1 a e f

Mark on intersections with axes and turning points

Just mark intersections with axes

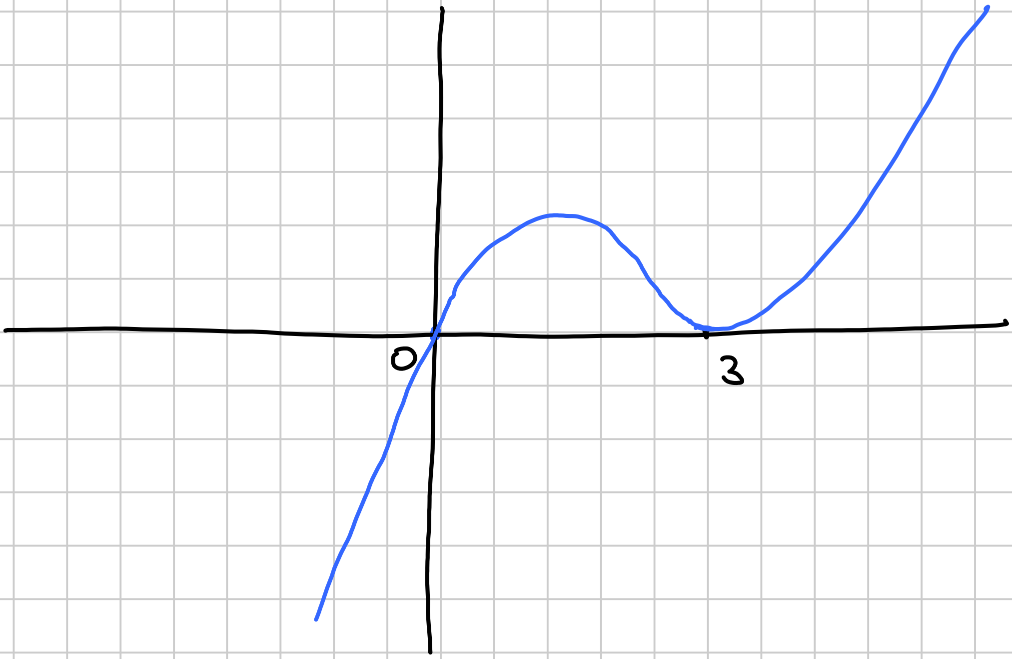
② $y = x^3 - 6x^2 + 9x$

When $x = 0$, $y = 0$

When $y = 0$,

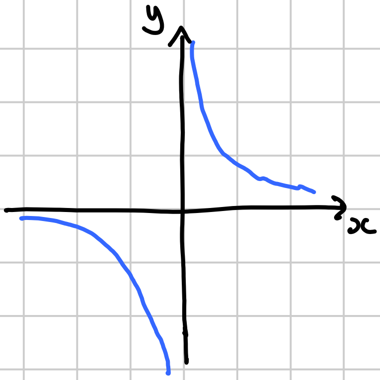
$$x^3 - 6x^2 + 9x = 0$$
$$x(x^2 - 6x + 9) = 0$$
$$x(x-3)(x-3) = 0$$
$$x = 0 \text{ or } x = 3 \text{ (repeated root)}$$

A double repeated root means that the graph just TOUCHES the axis ie the axis is a TANGENT to the graph

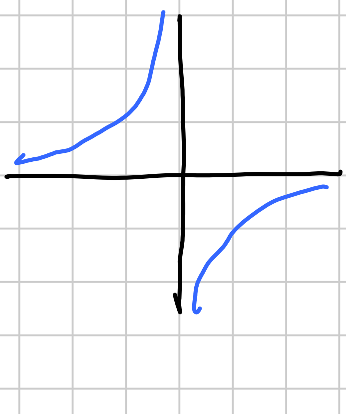


Reciprocal Graphs

The graph of $y = \frac{k}{x}$ looks like



if k is positive.
The x and y axes are ASYMPTOTES.



if k is negative

Function Notation and Transformations of Graphs

Examples of notation

1. If $f(x) = x^2$

(a) $f(-7) = 49$

(b) $f(t) = t^2$

(c) $f(x+4) = (x+4)^2$

(d) $f(x) + 4 = x^2 + 4$

(e) $f(-x) = (-x)^2 = x^2$

(f) $-f(x) = -x^2$

(g) $f(3x) = (3x)^2 = 9x^2$

(h) $3f(x) = 3x^2$

2. If $f(x) = \frac{1}{x}$

(a) $f(x-5) = \frac{1}{x-5}$

(b) $f(x) - 5 = \frac{1}{x} - 5$

(c) $f(4x) = \frac{1}{4x}$

(d) $4f(x) = 4 \times \frac{1}{x} = \frac{4}{x}$

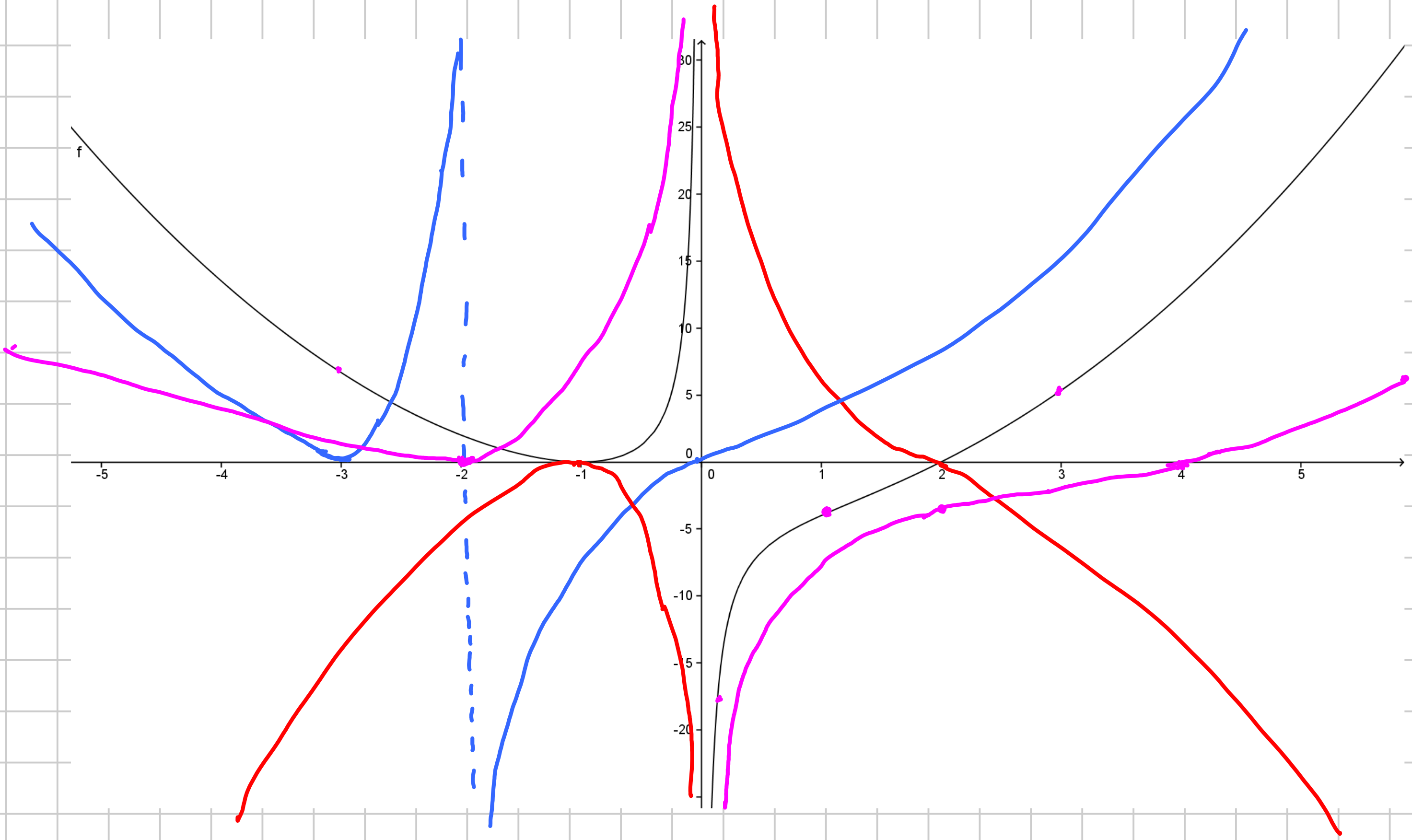
3. If $f(x) = x^3 - 4x^2 + x - 5$

(a) $f(x+2) = (x+2)^3 - 4(x+2)^2 + (x+2) - 5$

(b) $f\left(\frac{1}{x}\right) = \left(\frac{1}{x}\right)^3 - 4\left(\frac{1}{x}\right)^2 + \frac{1}{x} - 5$

p42 Ex 4A Q 2 begh, 3 adef
p47 Ex 4C Q 2

[For rules on transformations of graphs, see printed sheet]



Examples

(1) Above is the graph of a function $y = f(x)$. On the same axes, sketch the following:-

(a) $f(x+2)$ (Translation 2 to the left)

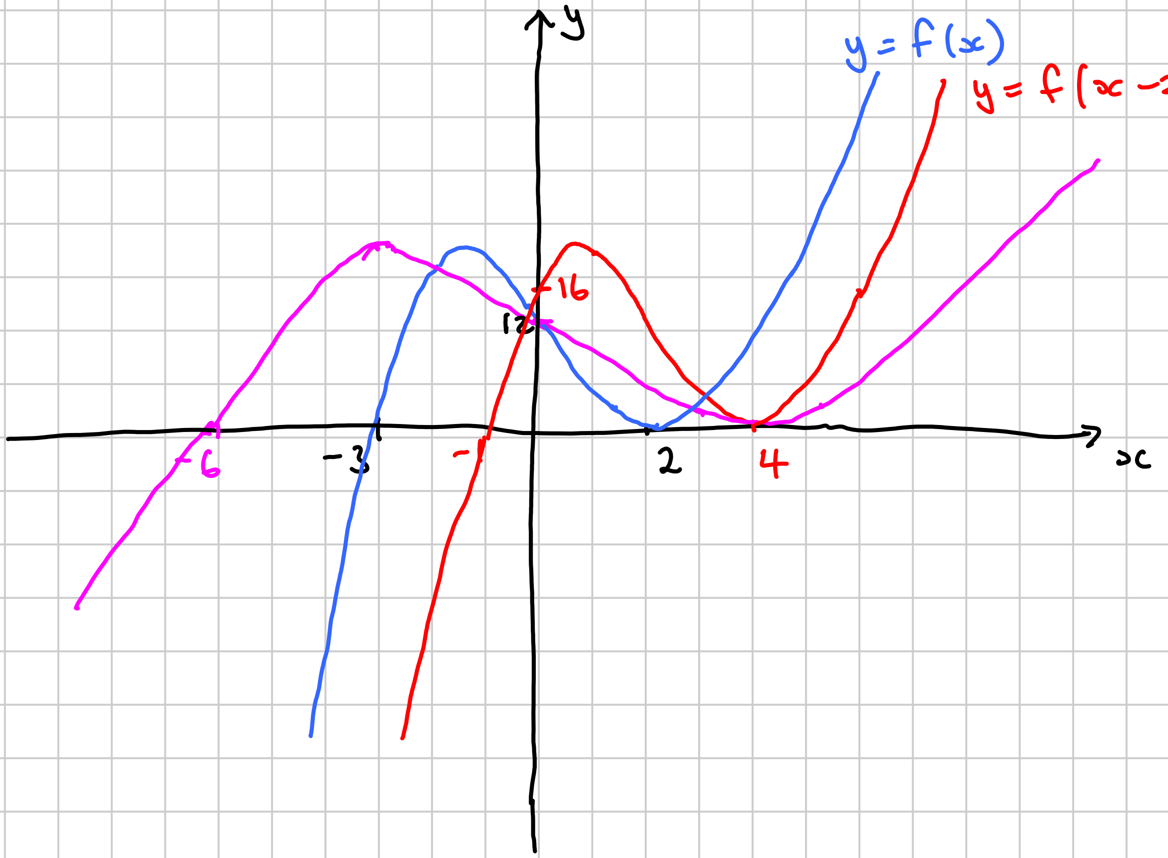
(b) $-f(x)$ (Reflection in the x-axis)

(c) $f(\frac{1}{2}x)$ (Stretch by factor 2 in x-axis)

p61 Ex 4G Q 1, 2, 3
(3 graphs per axes maximum!)

② Given that $f(x) = (x+3)(x-2)^2$

(a) Sketch the graph of $y = f(x)$. See ~ below



If $x = 0$, $y = 3 \times (-2)^2 = 12$

If $y = 0$, $x = -3$ or $x = 2$ (repeated root so touches axis here)

(b) Sketch $y = f(x-2)$. Translation 2 to right. See ~ above

Write down the equation of this graph and use it to find the y-intercept.

$$f(\;\;) = (\;\;+3)(\;\;-2)^2$$

$$f(x-2) = (x-2+3)(\;\;\;\;-2)^2$$

$$= (x+1)(x-4)^2$$

If $x = 0$, $y = (1)(-4)^2 = 16$

(c) Sketch $y = f\left(\frac{1}{2}x\right)$ "Stretch" by factor $\frac{1}{2}$ (ie a squash) in the x-direction

See ~ above