

COORDINATE GEOMETRY

Note Title

06/10/2011

Summary

If we have two points $A(x_1, y_1)$ and $B(x_2, y_2)$,

- The midpoint of AB is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
- The gradient of AB is $\frac{y_2 - y_1}{x_2 - x_1}$
- The distance AB is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

If we have a line with gradient m and y -intercept c , its equation is $y = mx + c$

If we have a line with gradient m , passing through (x_1, y_1) its equation is

$$y - y_1 = m(x - x_1)$$

If two lines with gradients m_1 and m_2 are perpendicular, then

$$m_1 m_2 = -1$$

or

$$m_2 = \frac{-1}{m_1}$$

Examples

- ① Find the equation of the line passing through the points $(-2, 5)$ and $(4, -4)$. Give your answer in the form $ax + by + c = 0$ where a, b and c are integers.

$$\text{Gradient} = \frac{5 - (-4)}{-2 - 4} = \frac{9}{-6} = -1\frac{1}{2}$$

Eqn

$$y - 5 = -1\frac{1}{2}(x - (-2))$$

$$y - 5 = -1\frac{1}{2}x + \frac{1}{2}(2 + 2)$$

$$y - 5 = -1\frac{1}{2}x - 3$$

(double both sides)

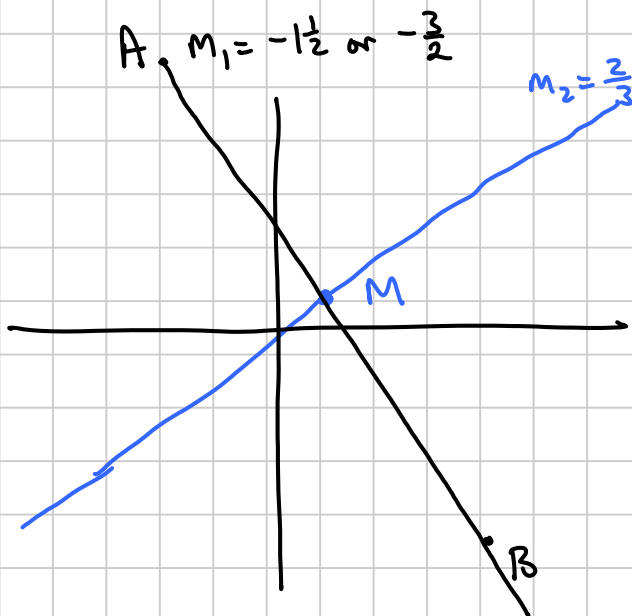
$$2y = -3x + 4$$

$$\underline{\underline{3x + 2y - 4 = 0}}$$

- ② Find the equation of the perpendicular bisector of the points A $(-2, 5)$ and B $(4, -4)$

Midpoint of A and B is $\left(\frac{-2+4}{2}, \frac{5-4}{2}\right)$

ie/ $\left(1, \frac{1}{2}\right)$



$$\text{Gradient of AB} = -1\frac{1}{2}$$

Gradient of perpendicular to AB

$$= \frac{-1}{-1\frac{1}{2}} = +\frac{2}{3}$$

Equation of perpendicular bisector:

$$y - \frac{1}{2} = \frac{2}{3}(x - 1)$$

$$y - \frac{1}{2} = \frac{2}{3}x - \frac{2}{3}$$

(multiply both sides by 6)

$$6y - 3 = 4x - 4$$

$$\underline{\underline{6y - 4x + 1 = 0}}$$

③ A (-2, 2), B (4, 5) and C (6, -4) are three points.

(a) Find the equation of the line joining A and B

$$\text{Gradient AB} = \frac{5-2}{4-(-2)} = \frac{3}{6} = \frac{1}{2}$$

$$\begin{aligned}\text{Line AB is } y - 2 &= \frac{1}{2}(x - (-2)) \\ y - 2 &= \frac{1}{2}x + 1 \\ \underline{y} &= \underline{\frac{1}{2}x + 3}\end{aligned}$$

(b) Find the equation of the line through C perpendicular to the line AB.

$$\text{Gradient of this line is } \frac{-1}{\frac{1}{2}} = -2$$

$$\begin{aligned}\text{Equation is } y - (-4) &= -2(x - 6) \\ y + 4 &= -2x + 12 \\ \underline{y} &= \underline{-2x + 8}\end{aligned}$$

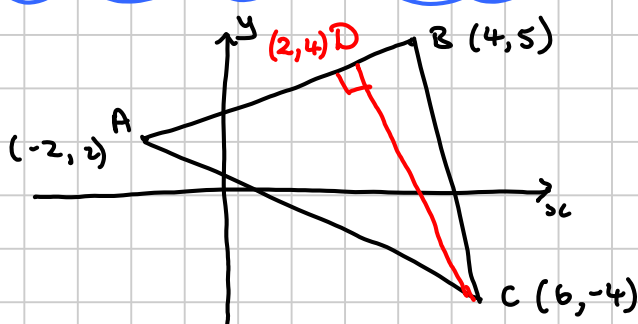
(c) The line in (b) meets the line AB at D. Find the coordinates of D.

$$\begin{aligned}\text{Solve simultaneously: } & y = \frac{1}{2}x + 3 \quad \textcircled{1} \\ & y = -2x + 8 \quad \textcircled{2} \\ \frac{1}{2}x + 3 &= -2x + 8 \\ 2\frac{1}{2}x &= 5 \\ x = 2 &\Rightarrow y = 4\end{aligned}$$

D is the point (2, 4)

(d) Hence find the area of triangle ABC in its simplest form

If in doubt, draw a diagram



$$\begin{aligned}\text{length AB} &= \sqrt{(4-(-2))^2 + (5-2)^2} \\ &= \sqrt{6^2 + 3^2} \\ &= \sqrt{45} = \sqrt{9 \times 5} = 3\sqrt{5}\end{aligned}$$

$$\begin{aligned}\text{length CD} &= \sqrt{8^2 + (-4)^2} \\ &= \sqrt{80} = \sqrt{16 \times 5} = 4\sqrt{5}\end{aligned}$$

$$\begin{aligned}\text{Area of } \triangle ABC &= \frac{1}{2} \times 3\sqrt{5} \times 4\sqrt{5} = 6 \times 5 \\ &= \underline{\underline{30 \text{ sq units}}}\end{aligned}$$

p 67 Ex 5A Q 1 hij, 2 hij, 3 hij, 4, 8, 10
p 69 Ex 5B Q 1 cgmn, 2, 3, 7, 9
p 71 Ex 5C Q 1 ae, 2, 4
p 74 Ex 5D Q 1 ch, 4, 5, 7, 8, 10
p 77 Ex 5E Q 1, 2, 3, 6, 7
[p 78 Ex 5F Q 1, 2, 3 if finished]

} Finish for HWK

(Also C2 p 51 Ex 4A Q 2, 5, 7, 10
p 56 Ex 4B Q 1, 5, 7, 9
p 59 Ex 4C Q 1 k, n, 2, 3, 5, 8, 10)