

More quadratic equations and inequalities

Note Title

13/09/2012

Simultaneous Equations one of which is quadratic

e.g. $x + 2y = -3$ ①
 $x^2 - 2x + 3y^2 = 11$ ②

In the LINEAR eqn, make x or y the subject.

$$x = -2y - 3$$

Substitute this into the QUADRATIC and solve it

$$(-2y - 3)^2 - 2(-2y - 3) + 3y^2 = 11$$

$$4y^2 + 12y + 9 + 4y + 6 + 3y^2 = 11$$

$$7y^2 + 16y + 15 = 11$$

$$7y^2 + 14y + 2y + 4 = 0$$

$$7y(y+2) + 2(y+2) = 0$$

$$(7y+2)(y+2) = 0$$

$$y = -\frac{2}{7} \quad \text{or} \quad y = -2$$

Substitute into LINEAR eqn to find corresponding values of x

$$\text{If } \underline{y = -\frac{2}{7}}, \quad x = \frac{4}{7} - 3 = \underline{-2\frac{3}{7}}$$

$$\text{If } \underline{y = -2}, \quad x = 4 - 3 = \underline{1}$$

Quadratic Inequalities

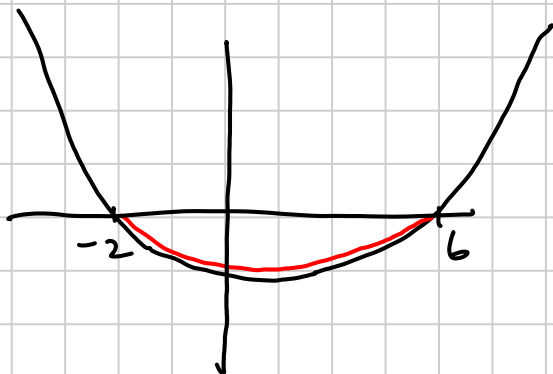
To solve these,

- write in the form $\text{LHS} < 0$ or $\text{LHS} > 0$
- sketch the graph $y = \text{LHS}$ showing the x -intercepts.
- Write down the set of values of x for which the graph is below the x -axis (for $\text{LHS} < 0$)
above the x -axis (for $\text{LHS} > 0$)

Examples

① $x^2 - 4x - 12 < 0$

$y = x^2 - 4x - 12$ crosses x -axis when $x^2 - 4x - 12 = 0$
 $(x+2)(x-6) = 0$
 $x = -2$ or $x = 6$



we want $y < 0$

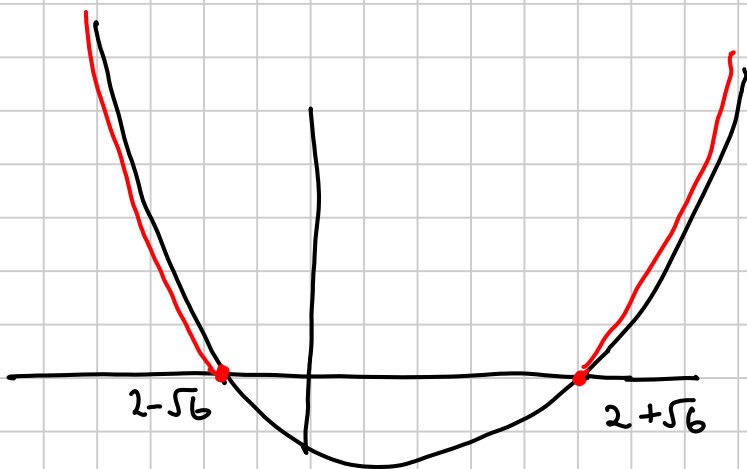
\Rightarrow $-2 < x < 6$

② $x^2 \geq 4x + 2$

$x^2 - 4x - 2 \geq 0$

$y = \text{LHS}$ crosses x -axis when $x^2 - 4x - 2 = 0$

$x^2 - 4x + 4 - 4 - 2 = 0$
 $(x-2)^2 - 6 = 0$
 $(x-2)^2 = 6$
 $x-2 = \pm\sqrt{6}$
 $x = 2 \pm \sqrt{6}$



$y \geq 0$ when $x \leq 2 - \sqrt{6}$ or $x \geq 2 + \sqrt{6}$

[Note that this cannot be written as a single inequality $a \leq x \leq b$ because it is two separate intervals.]

Simultaneous Inequalities

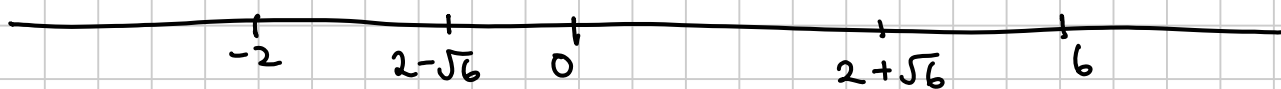
These are best solved using a number line.

Example Find the set of values of x for which

$$2 \leq x^2 - 4x < 12$$

This is two inequalities :-

- ① $x^2 - 4x < 12$ (see eg 1) $\Rightarrow -2 < x < 6$
- ② $x^2 - 4x \geq 2$ (see eg 2) $\Rightarrow x \leq 2 - \sqrt{6}$
or $x \geq 2 + \sqrt{6}$



①



②



Solution is $-2 < x \leq 2 - \sqrt{6}$ or $2 + \sqrt{6} \leq x < 6$

p 54 Ex 4.1 Q 2f, 3f, 4c

p 57 Ex 4.2 Q 1e, 2adf, 4, 5a, 6bc