EQUATIONS AND INEQUALITIES

Note Title 11/10/2011

Simultaneous Equations

A LINEAR equation does not contain any powers, algebraic fractions etc: e.g $y = 3x + 7$ or $2x - 3y = 5$.
Anything else is NON-LINEAR : e.g $x - y^2 = 8$, $\frac{3}{x} + y = 7$.

We usually solve LINEAR simultaneous equations by the elimination method, like we did at GCSE.

If we have one linear and one NON-LINEAR equation, we use the SUBSTITUTION method:

- Make y (or x) the subject of the LINEAR equation.
- Substitute this into the NON-LINEAR equation
- Solve this equation (there may be more than one solution)
- Use the linear equation to find the second variable for each solution.

Example

$$2x + y = 5 \quad (1)$$
$$2x^2 - y^2 = 17 \quad (2)$$

From (1), $y = 5 - 2x$

Subst into (2)

$$2x^2 - (5 - 2x)^2 = 17$$
$$2x^2 - (25 - 20x + 4x^2) = 17$$
$$-2x^2 + 20x - 25 = 17$$
$$0 = 2x^2 - 20x + 42$$
$$0 = x^2 - 10x + 21$$
$$0 = (x - 3)(x - 7)$$

$x = 3$ or $x = 7$

Subst into (1):

If $x = 3$, $y = 5 - 2x = -1$
If $x = 7$, $y = -9$

(We can check that these work in eqn (2))

p 28 Ex 3C Q 1 abd, 2

Inequalities

We use the same methods as for equations except that

> If we multiply or divide both sides by a negative number, we change the direction of the inequalities

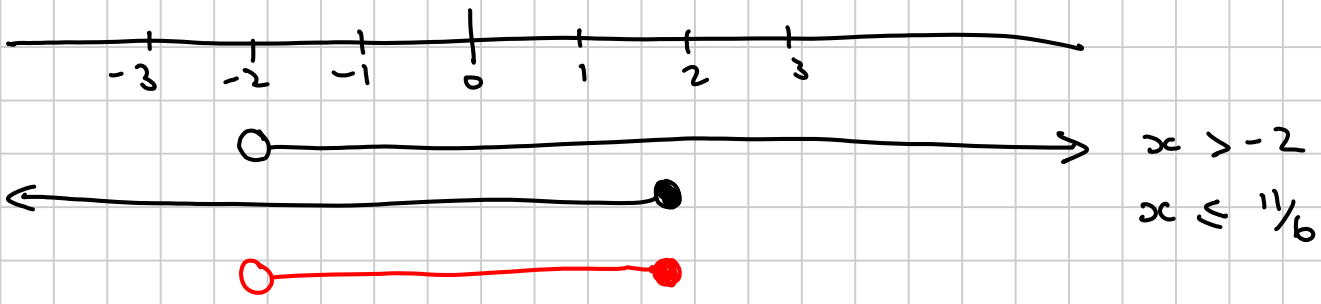
Example Find the set of values of x for which

$$13 - 3x < 19$$
$$\text{and} \quad 6x - 5 \leq 6$$

$$13 - 3x < 19$$
$$(-13) \qquad (-13)$$
$$-3x < 6$$
$$(\div -3) \qquad (\div -3)$$
$$x > -2 \quad ①$$

(dividing by -3, so change direction of inequality)

$$6x - 5 \leq 6$$
$$6x \leq 11$$
$$x \leq \frac{11}{6} \quad ②$$



Solution to both inequalities is $-2 < x \leq \frac{11}{6}$

Quadratic Inequalities

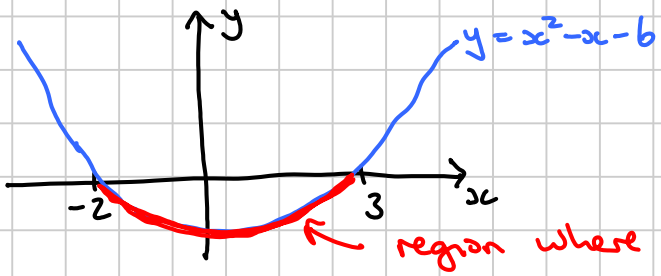
To solve these :-

- Rearrange so that RHS is 0.
- Sketch the graph of $y =$ LHS, showing intersections with the x-axis
- For < 0, look for part of graph below x-axis
  For > 0 " " " " " above "
- For a single region, write one 'double-ended' inequality
  For two separate regions, write two inequalities.

Examples

① $x^2 < x + 6$

$x^2 - x - 6 < 0$



If $y = 0$

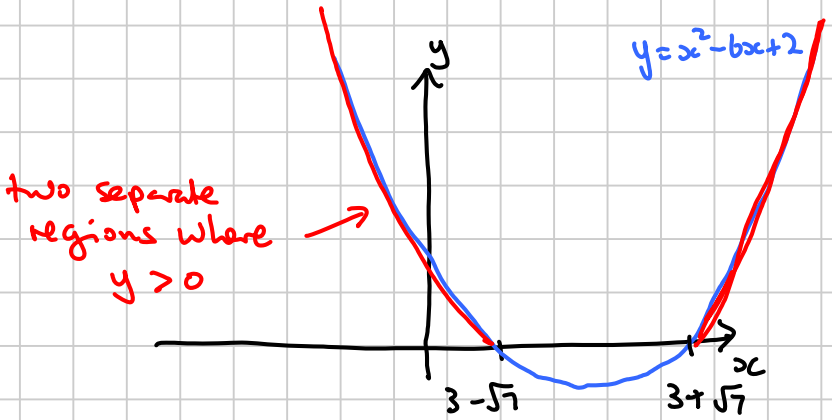
$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$x = 3$ or $x = -2$

Solution $-2 < x < 3$

② $x^2 - 6x + 2 > 0$



If $x^2 - 6x + 2 = 0$

$(x-3)^2 - 7 = 0$

$(x-3)^2 = 7$

$x - 3 = \pm\sqrt{7}$

$x = 3 \pm \sqrt{7}$

Solution is $x < 3 - \sqrt{7}$ or $x > 3 + \sqrt{7}$

Ex 3C Q 1 abd, 2

p 31 Ex 3D Q 3 abd

p 35 Ex 3E Q 1 ac, 2 ab, 3 ab

see p35 Example 12 if stuck.